

The Basketball And Tennis Ball Problem

If the algebra in the 1D handout seemed a little like pulling rabbits out of a hat, here is a more obvious but more laborious version:

Assume m_1 , v_1 , m_2 , and v_2 are given. Let v_3 = final velocity of m_1 , and v_4 = final velocity of m_2 . There are two unknowns (v_3 , v_4), so we need two equations. Conservation of momentum and kinetic energy give us:

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4 \quad \text{and} \quad \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_3^2 + \frac{1}{2} m_2 v_4^2$$

We make our life easier by defining $f = m_1/m_2$. If we divide through both equations by m_2 we can toss away a lot of subscripts, and we have $fv_1 + v_2 = fv_3 + v_4$ and $fv_1^2 + v_2^2 = fv_3^2 + v_4^2$. (We have also divided out the $\frac{1}{2}$ in the kinetic energy.) Solving the first equation for v_4 and inserting it into the second equation yields:

$$fv_1^2 + v_2^2 = fv_3^2 + (fv_1 + v_2 - fv_3)^2$$

Expanding the right-hand side:

$$fv_1^2 + v_2^2 = fv_3^2 + f^2 v_1^2 + v_2^2 + f^2 v_3^2 - 2f^2 v_1 v_3 - 2fv_2 v_3 + 2fv_1 v_2$$

Cancelling out the v_2^2 on each side and dividing through by f :

$$v_1^2 = v_3^2 - 2fv_1 v_3 - 2v_2 v_3 + fv_3^2 + 2v_1 v_2 + fv_1^2$$

Collecting terms:

$$(1 + f)v_3^2 - 2(fv_1 + v_2)v_3 + (2v_1 v_2 - v_1^2 + fv_1^2) = 0$$

This is a quadratic equation, and writing out the solution gives us:

$$v_3 = \{2(fv_1 + v_2) \pm [4(fv_1 + v_2)^2 - 4(1 + f)(2v_1 v_2 - v_1^2 + fv_1^2)]^{1/2}\} / 2(1 + f)$$

Dividing out factors of 2 and expanding the terms under the square root:

$$(1 + f)v_3 = (fv_1 + v_2) \pm [f^2 v_1^2 + v_2^2 + 2fv_1 v_2 - 2v_1 v_2 + v_1^2 - fv_1^2 - 2fv_1 v_2 + fv_1^2 - f^2 v_1^2]^{1/2}$$

Cancelling terms:

$$(1 + f)v_3 = (fv_1 + v_2) \pm [v_2^2 - 2v_1 v_2 + v_1^2]^{1/2} = (fv_1 + v_2) \pm (v_1 - v_2)$$

Which gives us our final solution of:

$v_3 = [(f - 1)v_1 + 2v_2]/(1 + f)$, where we have chosen the negative sign from the “ \pm ” because the plus sign yields the trivial solution $v_3 = v_1$, which means there was no collision. v_4 can be obtained by substituting the solution for v_3 back into the original conservation of momentum equation.

Now, let's apply our solution for v_3 to the case of the rebounding tennis ball on top of a basketball, which I demonstrated in class. A tennis ball has a mass of 0.056 kg, and a basketball has a mass of 0.60 kg, so this gives us $f = 0.056/0.60 = 0.093$, which I will round off to one-tenth. Since the two balls are dropped from the same height, they have the same speed when they hit the floor. However, a split-second after impact the basketball is headed in the opposite direction from the tennis ball, so $v_2 = -v_1$. Putting this into the solution for v_3 gives us $v_3 = [-0.9 v_1 + 2(-v_1)] / 1.1 = -2.6 v_1$. The negative sign just means that the tennis ball was moving downward at the moment of impact, but is moving upward after impact. The speed of the tennis ball has been increased by a factor of 2.6, so its kinetic energy has been increased by a factor of $2.6^2 = 6.8$, thus in principle it could bounce back to $h = 6.8 - 1 = 5.8$ times its original height.

The best I've ever seen the tennis ball do is maybe three times the original height, from which we can deduce that basketballs and tennis balls are not perfectly elastic. One of these days, I shall have to write a letter of protest to the manufacturers of these obviously defective items.