**6.30.** **IDENTIFY:** The magnitude of the work can be found by finding the area under the graph.

**SET UP:** The area under each triangle is 1/2 base$\times$height . $F_x > 0$ , so the work done is positive when $x$ increases during the displacement.

**EXECUTE:** **(a)** $1/2\,(8\text{ m})(10\text{ N}) = 40\text{ J}$ .

**(b)** $1/2\,(4\text{ m})(10\text{ N}) = 20\text{ J}$ .

**(c)** $1/2\,(12\text{ m})(10\text{ N}) = 60\text{ J}$ .

**EVALUATE:** The sum of the answers to parts (a) and (b) equals the answer to part (c).

**6.31.** **IDENTIFY:** Use the work-energy theorem and the results of Problem 6.30.

**SET UP:** For $x = 0$ to $x = 8.0\text{ m}$ , $W_{\text{tot}} = 40\text{ J}$ . For $x = 0$ to $x = 12.0\text{ m}$ , $W_{\text{tot}} = 60\text{ J}$ .

**EXECUTE:** **(a)** $v = \sqrt{\dfrac{(2)(40\text{ J})}{10\text{ kg}}} = 2.83\text{ m/s}$

**(b)** $v = \sqrt{\dfrac{(2)(60\text{ J})}{10\text{ kg}}} = 3.46\text{ m/s}$ .

**EVALUATE:** $\vec{F}$ is always in the $+x$-direction. For this motion $\vec{F}$ does positive work and the speed continually increases during the motion.

**6.39.** **IDENTIFY** and **SET UP:** Apply Eq.(6.6). Let point 1 be where the sled is released and point 2 be at $x = 0$ for part (a) and at $x = -0.200\text{ m}$ for part (b). Use Eq.(6.10) for the work done by the spring and calculate $K_2$. Then $K_2 = \tfrac{1}{2}mv_2^2$ gives $v_2$.

**EXECUTE:** **(a)** $W_{\text{tot}} = K_2 - K_1$ so $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$ (released with no initial velocity), $K_2 = \tfrac{1}{2}mv_2^2$

The only force doing work is the spring force. Eq.(6.10) gives the work done *on* the spring to move its end from $x_1$ to $x_2$. The force the spring exerts on an object attached to it is $F = -kx$, so the work the spring does is

$W_{\text{spr}} = -\left(\tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2\right) = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$. Here $x_1 = -0.375\text{ m}$ and $x_2 = 0$. Thus $W_{\text{spr}} = \tfrac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - 0 = 281\text{ J}$.

$K_2 = K_1 + W_{\text{tot}} = 0 + 281\text{ J} = 281\text{ J}$

Then $K_2 = \tfrac{1}{2}mv_2^2$ implies $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(281\text{ J})}{70.0\text{ kg}}} = 2.83\text{ m/s}$.

**(b)** $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$

$W_{\text{tot}} = W_{\text{spr}} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$. Now $x_2 = 0.200\text{ m}$, so

$W_{\text{spr}} = \tfrac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - \tfrac{1}{2}(4000\text{ N/m})(-0.200\text{ m})^2 = 281\text{ J} - 80\text{ J} = 201\text{ J}$

Thus $K_2 = 0 + 201\text{ J} = 201\text{ J}$ and $K_2 = \tfrac{1}{2}mv_2^2$ gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(201\text{ J})}{70.0\text{ kg}}} = 2.40\text{ m/s}$.

**EVALUATE:** The spring does positive work and the sled gains speed as it returns to $x = 0$. More work is done during the larger displacement in part (a), so the speed there is larger than in part (b).

**6.46.** **IDENTIFY:** The thermal energy is produced as a result of the force of friction, $F = \mu_k mg$. The average thermal power is thus the average rate of work done by friction or $P = F_{\|}v_{\text{av}}$ .

**SET UP:** $v_{\text{av}} = \dfrac{v_2 + v_1}{2} = \left(\dfrac{8.00\text{ m/s} + 0}{2}\right) = 4.00\text{ m/s}$

**EXECUTE:** $P = Fv_{\text{av}} = \left[(0.200)(20.0\text{ kg})(9.80\text{ m/s}^2)\right](4.00\text{ m/s}) = 157\text{ W}$

**EVALUATE:** The power could also be determined as the rate of change of kinetic energy, $\Delta K/t$, where the time is calculated from $v_f = v_i + at$ and $a$ is calculated from a force balance, $\sum F = ma = \mu_k mg$.

**6.49.** **IDENTIFY:** $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$ . The work you do in lifting mass $m$ a height $h$ is $mgh$.

**SET UP:** $1\text{ hp} = 746\text{ W}$

**EXECUTE:** **(a)** The number per minute would be the average power divided by the work ($mgh$) required to lift one box, $\frac{(0.50 \text{ hp})(746 \text{ W/hp})}{(30 \text{ kg})(9.80 \text{ m/s}^2)(0.90 \text{ m})} = 1.41\text{ /s, or } 84.6\text{ /min.}$

**(b)** Similarly, $\frac{(100 \text{ W})}{(30 \text{ kg})(9.80 \text{ m/s}^2)(0.90 \text{ m})} = 0.378\text{ /s, or } 22.7\text{ /min.}$

**EVALUATE:** A 30-kg crate weighs about 66 lbs. It is not possible for a person to perform work at this rate.

**6.67.** **IDENTIFY:** Calculate the work done by friction and apply $W_{tot} = K_2 - K_1$. Since the friction force is not constant, use Eq.(6.7) to calculate the work.

**SET UP:** Let $x$ be the distance past $P$. Since $\mu_k$ increases linearly with $x$, $\mu_k = 0.100 + Ax$. When $x = 12.5$ m, $\mu_k = 0.600$, so $A = 0.500/(12.5 \text{ m}) = 0.0400/\text{m}$

**EXECUTE:** **(a)** $W_{tot} = \Delta K = K_2 - K_1$ gives $-\int \mu_k mg dx = 0 - \frac{1}{2}mv_1^2$. Using the above expression for $\mu_k$,

$g\int_0^{x_2}(0.100 + Ax)dx = \frac{1}{2}v_1^2$ and $g\left[(0.100)x_2 + A\frac{x_2^2}{2}\right] = \frac{1}{2}v_1^2$. $(9.80 \text{ m/s}^2)\left[(0.100)x_f + (0.0400/\text{m})\frac{x_f^2}{2}\right] = \frac{1}{2}(4.50 \text{ m/s})^2$.

Solving for $x_2$ gives $x_2 = 5.11$ m.

**(b)** $\mu_k = 0.100 + (0.0400/\text{m})(5.11 \text{ m}) = 0.304$

**(c)** $W_{tot} = K_2 - K_1$ gives $-\mu_k mgx_2 = 0 - \frac{1}{2}mv_1^2$. $x_2 = \frac{v_1^2}{2\mu_k g} = \frac{(4.50 \text{ m/s})^2}{2(0.100)(9.80 \text{ m/s}^2)} = 10.3$ m.

**EVALUATE:** The box goes farther when the friction coefficient doesn't increase.

**6.68.** **IDENTIFY:** Use Eq.(6.7) to calculate $W$.

**SET UP:** $x_1 = 0$. In part (a), $x_2 = 0.050$ m. In part (b), $x_2 = -0.050$ m.

**EXECUTE:** **(a)** $W = \int_0^{x_2} F dx = \int_0^{x_2}(kx - bx^2 + cx^3)dx = \frac{k}{2}x_2^2 - \frac{b}{3}x_2^3 + \frac{c}{4}x_2^4$.

$W = (50.0 \text{ N/m})x_2^2 - (233 \text{ N/m}^2)x_2^3 + (3000 \text{ N/m}^3)x_2^4$. When $x_2 = 0.050$ m, $W = 0.12$ J.

**(b)** When $x_2 = -0.050$ m, $W = 0.17$ J.

**(c)** It's easier to stretch the spring; the quadratic $-bx^2$ term is always in the $-x$-direction, and so the needed force, and hence the needed work, will be less when $x_2 > 0$.

**EVALUATE:** When $x = 0.050$ m, $F_x = 4.75$ N. When $x = -0.050$ m, $F_x = 8.25$ N.

**6.80.** **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$. $W = Fs\cos\phi$.

**SET UP:** The students do positive work, and the force that they exert makes an angle of $30.0°$ with the direction of motion. Gravity does negative work, and is at an angle of $120.0°$ with the chair's motion,

**EXECUTE:** The total work done is $W_{tot} = ((600 \text{ N})\cos 30.0° + (85.0 \text{ kg})(9.80 \text{ m/s}^2)\cos 120.0°)(2.50 \text{ m}) = 257.8$ J,

and so the speed at the top of the ramp is $v_2 = \sqrt{v_1^2 + \frac{2W_{tot}}{m}} = \sqrt{(2.00 \text{ m/s})^2 + \frac{2(257.8 \text{ J})}{(85.0 \text{ kg})}} = 3.17$ m/s.

**EVALUATE:** The component of gravity down the incline is $mg\sin 30° = 417$ N and the component of the push up the incline is $(600 \text{ N})\cos 30° = 520$ N. The force component up the incline is greater than the force component down the incline, the net work done is positive and the speed increases.

**6.95.** **IDENTIFY:** $P = F_{\parallel}v$. The force required to give mass $m$ an acceleration $a$ is $F = ma$. For an incline at an angle $\alpha$ above the horizontal, the component of $mg$ down the incline is $mg\sin\alpha$.

**SET UP:** For small $\alpha$, $\sin\alpha \approx \tan\alpha$.

**EXECUTE:** **(a)** $P_0 = Fv = (53\times10^3 \text{ N})(45 \text{ m/s}) = 2.4$ MW.

**(b)** $P_1 = mav = (9.1\times10^5 \text{ kg})(1.5 \text{ m/s}^2)(45 \text{ m/s}) = 61$ MW.

**(c)** Approximating $\sin\alpha$, by $\tan\alpha$, and using the component of gravity down the incline as $mg\sin\alpha$,

$P_2 = (mg\sin\alpha)v = (9.1\times10^5 \text{ kg})(9.80 \text{ m/s}^2)(0.015)(45 \text{ m/s}) = 6.0$ MW.

**EVALUATE:** From Problem 6.94, we would expect that a $0.15 \text{ m/s}^2$ acceleration and a 1.5% slope would require the same power. We found that a $1.5 \text{ m/s}^2$ acceleration requires ten times more power than a 1.5% slope, which is consistent.