7.5 **IDENTIFY** and **SET UP:** Use energy methods.

(a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Solve for K_2 and then use $K_2 = \frac{1}{2}mv_2^2$ to obtain v_2.

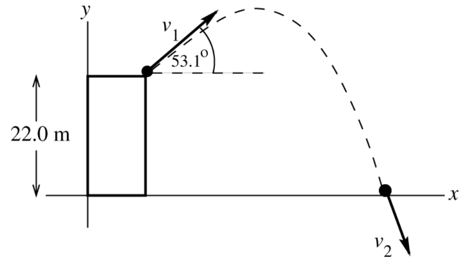

Figure 7.5

$W_{\text{other}} = 0$ (The only force on the ball while it is in the air is gravity.)
$K_1 = \frac{1}{2}mv_1^2$; $K_2 = \frac{1}{2}mv_2^2$
$U_1 = mgy_1$, $y_1 = 22.0$ m
$U_2 = mgy_2 = 0$, since $y_2 = 0$ for our choice of coordinates.

EXECUTE: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(12.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(22.0\text{ m})} = 24.0\text{ m/s}$$

EVALUATE: The projection angle of $53.1°$ doesn't enter into the calculation. The kinetic energy depends only on the magnitude of the velocity; it is independent of the direction of the velocity.

(b) Nothing changes in the calculation. The expression derived in part (a) for v_2 is independent of the angle, so $v_2 = 24.0$ m/s, the same as in part (a).

(c) The ball travels a shorter distance in part (b), so in that case air resistance will have less effect.

7.12 **IDENTIFY:** Only gravity does work, so apply Eq.(7.5).

SET UP: $v_1 = 0$, so $\frac{1}{2}mv_2^2 = mg(y_1 - y_2)$.

EXECUTE: Tarzan is lower than his original height by a distance $y_1 - y_2 = l(\cos 30° - \cos 45°)$ so his speed is $v = \sqrt{2gl(\cos 30° - \cos 45°)} = 7.9$ m/s, a bit quick for conversation.

EVALUATE: The result is independent of Tarzan's mass.

7.13

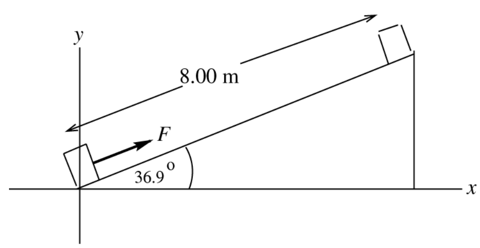

Figure 7.13a

$y_1 = 0$
$y_2 = (8.00\text{ m})\sin 36.9°$
$y_2 = 4.80$ m

(a) IDENTIFY and **SET UP:** $\vec{F}$ is constant so Eq.(6.2) can be used. The situation is sketched in Figure 7.13a.

EXECUTE: $W_F = (F\cos\phi)s = (110\text{ N})(\cos 0°)(8.00\text{ m}) = 880$ J

EVALUATE: $\vec{F}$ is in the direction of the displacement and does positive work.

(b) IDENTIFY and **SET UP:** Calculate W using Eq.(6.2) but first must calculate the friction force. Use the free-body diagram for the oven sketched in Figure 7.13b to calculate the normal force n; then the friction force can be calculated from $f_k = \mu_k n$. For this calculation use coordinates parallel and perpendicular to the incline.

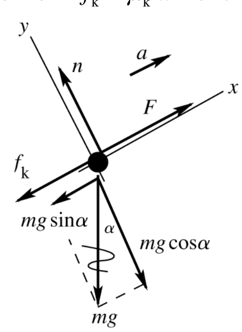

Figure 7.13b

EXECUTE: $\sum F_y = ma_y$
$n - mg\cos 36.9° = 0$
$n = mg\cos 36.9°$
$f_k = \mu_k n = \mu_k mg\cos 36.9°$
$f_k = (0.25)(10.0\text{ kg})(9.80\text{ m/s}^2)\cos 36.9° = 19.6$ N

$W_f = (f_k\cos\phi)s = (19.6\text{ N})(\cos 180°)(8.00\text{ m}) = -157$ J

EVALUATE: Friction does negative work.

(c) IDENTIFY and **SET UP:** $U = mgy$; take $y = 0$ at the bottom of the ramp.

EXECUTE: $\Delta U = U_2 - U_1 = mg(y_2 - y_1) = (10.0\text{ kg})(9.80\text{ m/s}^2)(4.80\text{ m} - 0) = 470\text{ J}$

EVALUATE: The object moves upward and U increases.

(d) IDENTIFY and **SET UP:** Use Eq.(7.7). Solve for ΔK.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$\Delta K = K_2 - K_1 = U_1 - U_2 + W_{\text{other}}$

$\Delta K = W_{\text{other}} - \Delta U$

$W_{\text{other}} = W_F + W_f = 880\text{ J} - 157\text{ J} = 723\text{ J}$

$\Delta U = 470\text{ J}$

Thus $\Delta K = 723\text{ J} - 470\text{ J} = 253\text{ J}$.

EVALUATE: W_{other} is positive. Some of W_{other} goes to increasing U and the rest goes to increasing K.

(e) IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the oven. Solve for $\vec{a}$ and then use a constant acceleration equation to calculate v_2.

SET UP: We can use the free-body diagram that is in part (b):

$\sum F_x = ma_x$

$F - f_k - mg\sin 36.9° = ma$

EXECUTE: $a = \dfrac{F - f_k - mg\sin 36.9°}{m} = \dfrac{110\text{ N} - 19.6\text{ N} - (10\text{ kg})(9.80\text{ m/s}^2)\sin 36.9°}{10.0\text{ kg}} = 3.16\text{ m/s}^2$

SET UP: $v_{1x} = 0$, $a_x = 3.16\text{ m/s}^2$, $x - x_0 = 8.00\text{ m}$, $v_{2x} = ?$

$v_{2x}^2 = v_{1x}^2 + 2a_x(x - x_0)$

EXECUTE: $v_{2x} = \sqrt{2a_x(x - x_0)} = \sqrt{2(3.16\text{ m/s}^2)(8.00\text{ m})} = 7.11\text{ m/s}^2$

Then $\Delta K = K_2 - K_1 = \frac{1}{2}mv_2^2 = \frac{1}{2}(10.0\text{ kg})(7.11\text{ m/s})^2 = 253\text{ J}$.

EVALUATE: This agrees with the result calculated in part (d) using energy methods.

7.25 **IDENTIFY:** Apply Eq.(7.13) and $F = ma$.

SET UP: $W_{\text{other}} = 0$. There is no change in U_{grav}. $K_1 = 0$, $U_2 = 0$.

EXECUTE: $\frac{1}{2}kx^2 = \frac{1}{2}mv_x^2$. The relations for m, v_x, k and x are $kx^2 = mv_x^2$ and $kx = 5mg$.

Dividing the first equation by the second gives $x = \dfrac{v_x^2}{5g}$, and substituting this into the second gives $k = 25\dfrac{mg^2}{v_x^2}$.

(a) $k = 25\dfrac{(1160\text{ kg})(9.80\text{ m/s}^2)^2}{(2.50\text{ m/s})^2} = 4.46\times 10^5\text{ N/m}$

(b) $x = \dfrac{(2.50\text{ m/s})^2}{5(9.80\text{ m/s}^2)} = 0.128\text{ m}$

EVALUATE: Our results for k and x do give the required values for a_x and v_x:

$$a_x = \frac{kx}{m} = \frac{(4.46\times 10^5\text{ N/m})(0.128\text{ m})}{1160\text{ kg}} = 49.2\text{ m/s}^2 = 5.0g \text{ and } v_x = x\sqrt{\frac{k}{m}} = 2.5\text{ m/s}.$$

7.39 **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the bag and to the box. Apply Eq.(7.7) to the motion of the system of the box and bucket after the bag is removed.

SET UP: Let $y = 0$ at the final height of the bucket, so $y_1 = 2.00\text{ m}$ and $y_2 = 0$. $K_1 = 0$. The box and the bucket move with the same speed v, so $K_2 = \frac{1}{2}(m_{\text{box}} + m_{\text{bucket}})v^2$. $W_{\text{other}} = -f_k d$, with $d = 2.00\text{ m}$ and $f_k = \mu_k m_{\text{box}} g$.

Before the bag is removed, the maximum possible friction force the roof can exert on the box is $(0.700)(80.0\text{ kg} + 50.0\text{ kg})(9.80\text{ m/s}^2) = 892\text{ N}$. This is larger than the weight of the bucket (637 N), so before the bag is removed the system is at rest.

EXECUTE: **(a)** The friction force on the bag of gravel is zero, since there is no other horizontal force on the bag for friction to oppose. The static friction force on the box equals the weight of the bucket, 637 N.

(b) Eq.(7.7) gives $m_{\text{bucket}} g y_1 - f_k d = \frac{1}{2} m_{\text{tot}} v^2$, with $m_{\text{tot}} = 145.0$ kg . $v = \sqrt{\frac{2}{m_{\text{tot}}}(m_{\text{bucket}} g y_1 - \mu_k m_{\text{box}} g d)}$.

$$v = \sqrt{\frac{2}{145.0\text{ kg}}\left[(65.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) - (0.400)(80.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})\right]}\ .$$

$v = 2.99$ m/s .

EVALUATE: If we apply $\sum \vec{F} = m\vec{a}$ to the box and to the bucket we can calculate their common acceleration *a*. Then a constant acceleration equation applied to either object gives $v = 2.99$ m/s , in agreement with our result obtained using energy methods.

7.45 **IDENTIFY:** At its highest point between bounces all the mechanical energy of the ball is in the form of gravitational potential energy.
SET UP: $E = U = mgh$, where *h* is the height at the highest point of the motion.

EXECUTE: **(a)** $mgh = (0.650\text{ kg})(9.80\text{ m/s}^2)(2.50\text{ m}) = 15.9$ J

(b) The second height is $0.75(2.50\text{ m}) = 1.875$ m, so the second $mgh = 11.9$ J ; it loses $15.9\text{ J} - 11.9\text{ J} = 4.0$ J on first bounce. This energy is converted to thermal energy.
(c) The third height is $0.75(1.875\text{ m}) = 1.40$ m, , so third $mgh = 8.9$ J ; it loses $11.9\text{ J} - 8.9\text{ J} = 3.0$ J on second bounce.
EVALUATE: In each bounce the ball loses 25% of its mechanical energy.

7.47 **(a) IDENTIFY:** Use work-energy relation to find the kinetic energy of the wood as it enters the rough bottom.
SET UP: Let point 1 be where the piece of wood is released and point 2 be just before it enters the rough bottom. Let $y = 0$ be at point 2.

EXECUTE: $U_1 = K_2$ gives $K_2 = mgy_1 = 78.4$ J.

IDENTIFY: Now apply work-energy relation to the motion along the rough bottom.
SET UP: Let point 1 be where it enters the rough bottom and point 2 be where it stops.

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

EXECUTE: $W_{\text{other}} = W_f = -\mu_k mgs,\quad K_2 = U_1 = U_2 = 0;\quad K_1 = 78.4$ J

$78.4\text{ J} - \mu_k mgs = 0;$ solving for *s* gives $s = 20.0$ m.

The wood stops after traveling 20.0 m along the rough bottom.
(b) Friction does -78.4 J of work.
EVALUATE: The piece of wood stops before it makes one trip across the rough bottom. The final mechanical energy is zero. The negative friction work takes away all the mechanical energy initially in the system.

7.56 **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the rocket from the starting point to the base of the ramp. W_{other} is the work done by the thrust and by friction.

SET UP: Let point 1 be at the starting point and let point 2 be at the base of the ramp. $v_1 = 0$, $v_2 = 50.0$ m/s . Let $y = 0$ at the base and take $+y$ upward. Then $y_2 = 0$ and $y_1 = d\sin 53°$, where *d* is the distance along the ramp from the base to the starting point. Friction does negative work.
EXECUTE: $K_1 = 0$, $U_2 = 0$. $U_1 + W_{\text{other}} = K_2$. $W_{\text{other}} = (2000\text{ N})d - (500\text{ N})d = (1500\text{ N})d$.

$mgd\sin 53° + (1500\text{ N})d = \frac{1}{2}mv_2^2$.

$$d = \frac{mv_2^2}{2[mg\sin 53° + 1500\text{ N}]} = \frac{(1500\text{ kg})(50.0\text{ m/s})^2}{2[(1500\text{ kg})(9.80\text{ m/s}^2)\sin 53° + 1500\text{ N}]} = 142\text{ m}\ .$$

EVALUATE: The initial height is $y_1 = (142\text{ m})\sin 53° = 113$ m . An object free-falling from this distance attains a speed $v = \sqrt{2gy_1} = 47.1$ m/s . The rocket attains a greater speed than this because the forward thrust is greater than the friction force.

7.59 **(a) IDENTIFY** and **SET UP:** Apply Eq.(7.7) to the motion of the potato.
Let point 1 be where the potato is released and point 2 be at the lowest point in its motion, as shown in Figure 7.59a.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

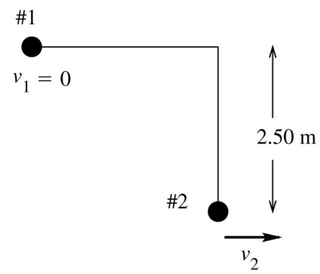

Figure 7.59a

$y_1 = 2.50$ m

$y_2 = 0$

The tension in the string is at all points in the motion perpendicular to the displacement, so $W_T = 0$

The only force that does work on the potato is gravity, so $W_{\text{other}} = 0$.

EXECUTE: $K_1 = 0,\ \ K_2 = \frac{1}{2}mv_2^2,\ \ U_1 = mgy_1,\ \ U_2 = 0$

Thus $U_1 = K_2$.

$mgy_1 = \frac{1}{2}mv_2^2$

$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(2.50\text{ m})} = 7.00\text{ m/s}$

EVALUATE: v_2 is the same as if the potato fell through 2.50 m.

(b) IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the potato. The potato moves in an arc of a circle so its acceleration is $\vec{a}_{\text{rad}}$, where $a_{\text{rad}} = v^2/R$ and is directed toward the center of the circle. Solve for one of the forces, the tension *T* in the string.

SET UP: The free-body diagram for the potato as it swings through its lowest point is given in Figure 7.59b.

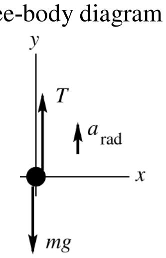

Figure 7.59b

The acceleration $\vec{a}_{\text{rad}}$ is directed in toward the center of the circular path, so at this point it is upward.

EXECUTE: $\sum F_y = ma_y$

$T - mg = ma_{\text{rad}}$

$T = m(g + a_{\text{rad}}) = m\left(g + \frac{v_2^2}{R}\right)$, where the radius *R* for the circular motion is the length *L* of the string.

It is instructive to use the algebraic expression for v_2 from part (a) rather than just putting in the numerical value:

$v_2 = \sqrt{2gy_1} = \sqrt{2gL}$, so $v_2^2 = 2gL$

Then $T = m\left(g + \frac{v_2^2}{L}\right) = m\left(g + \frac{2gL}{L}\right) = 3mg$; the tension at this point is three times the weight of the potato.

$$T = 3mg = 3(0.100\text{ kg})(9.80\text{ m/s}^2) = 2.94\text{ N}$$

EVALUATE: The tension is greater than the weight; the acceleration is upward so the net force must be upward.

7.67 $F_x = -\alpha x - \beta x^2,\ \ \alpha = 60.0\text{ N/m}$ and $\beta = 18.0\text{ N/m}^2$

(a) IDENTIFY: Use Eq.(6.7) to calculate *W* and then use $W = -\Delta U$ to identify the potential energy function $U(x)$.

SET UP: $W_{F_x} = U_1 - U_2 = \int_{x_1}^{x_2} F_x(x)\,dx$

Let $x_1 = 0$ and $U_1 = 0$. Let x_2 be some arbitrary point *x*, so $U_2 = U(x)$.

EXECUTE: $U(x) = -\int_0^x F_x(x)\,dx = -\int_0^x (-\alpha x - \beta x^2)\,dx = \int_0^x (\alpha x + \beta x^2)\,dx = \frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3$.

EVALUATE: If $\beta = 0$, the spring does obey Hooke's law, with $k = \alpha$, and our result reduces to $\frac{1}{2}kx^2$.

(b) IDENTIFY: Apply Eq.(7.15) to the motion of the object.

SET UP: The system at points 1 and 2 is sketched in Figure 7.67.

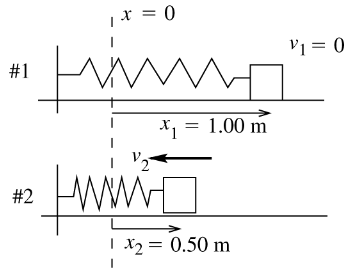

Figure 7.67

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$
The only force that does work on the object is the spring force, so $W_{\text{other}} = 0$.

EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$U_1 = U(x_1) = \frac{1}{2}\alpha x_1^2 + \frac{1}{3}\beta x_1^3 = \frac{1}{2}(60.0\text{ N/m})(1.00\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(1.00\text{ m})^3 = 36.0\text{ J}$

$U_2 = U(x_2) = \frac{1}{2}\alpha x_2^2 + \frac{1}{3}\beta x_2^3 = \frac{1}{2}(60.0\text{ N/m})(0.500\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(0.500\text{ m})^3 = 8.25\text{ J}$

Thus $36.0\text{ J} = \frac{1}{2}mv_2^2 + 8.25\text{ J}$

$$v_2 = \sqrt{\frac{2(36.0\text{ J} - 8.25\text{ J})}{0.900\text{ kg}}} = 7.85\text{ m/s}$$

EVALUATE: The elastic potential energy stored in the spring decreases and the kinetic energy of the object increases.

7.76 **IDENTIFY:** Apply Eq.(7.14) to the motion of the student.

SET UP: Let $x_0 = 0.18\text{ m}$, $x_1 = 0.71\text{ m}$. The spring constants (assumed identical) are then known in terms of the unknown weight *w*, $4kx_0 = w$. Let $y = 0$ at the initial position of the student.

EXECUTE: **(a)** The speed of the brother at a given height *h* above the point of maximum compression is then found from $\frac{1}{2}(4k)x_1^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 + mgh$, or $v^2 = \frac{(4k)g}{w}x_1^2 - 2gh = g\left(\frac{x_1^2}{x_0} - 2h\right)$. Therefore,

$v = \sqrt{(9.80\text{ m/s}^2)((0.71\text{ m})^2/(0.18\text{ m}) - 2(0.90\text{ m}))} = 3.13\text{ m/s}$, or 3.1 m/s to two figures.

(b) Setting $v = 0$ and solving for *h*, $h = \frac{2kx_1^2}{mg} = \frac{x_1^2}{2x_0} = 1.40\text{ m}$, or 1.4 m to two figures.

(c) No; the distance x_0 will be different, and the ratio $\frac{x_1^2}{x_0} = \frac{(x_1 + 0.53\text{ m})^2}{x_1} = x_1\left(1 + \frac{0.53\text{ m}}{x_1}\right)^2$ will be different.

Note that on a planet with lower *g*, x_1 will be smaller and *h* will be larger.

EVALUATE: We are able to solve the problem without knowing either the mass of the student or the force constant of the spring.