8.50 **IDENTIFY:** Apply Eqs. 8.28, 8.30 and 8.32. There is only one component of position and velocity.

SET UP: $m_A = 1200\text{ kg}$, $m_B = 1800\text{ kg}$. $M = m_A + m_B = 3000\text{ kg}$. Let +*x* be to the right and let the origin be at the center of mass of the station wagon.

EXECUTE: **(a)** $x_{\text{cm}} = \dfrac{m_A x_A + m_B x_B}{m_A + m_B} = \dfrac{0 + (1800\text{ kg})(40.0\text{ m})}{1200\text{ kg} + 1800\text{ kg}} = 24.0\text{ m}.$

The center of mass is between the two cars, 24.0 m to the right of the station wagon and 16.0 m behind the lead car.

(b) $P_x = m_A v_{A1} + m_B v_{B1} = (1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s}) = 5.04\times10^4\text{ kg}\cdot\text{m/s}.$

(c) $v_{\text{cm},x} = \dfrac{m_A v_{A,x} + m_B v_{B,x}}{m_A + m_B} = \dfrac{(1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s})}{1200\text{ kg} + 1800\text{ kg}} = 16.8\text{ m/s}.$

(d) $P_x = Mv_{\text{cm-}x} = (3000\text{ kg})(16.8\text{ m/s}) = 5.04\times10^4\text{ kg}\cdot\text{m/s}$, the same as in part (b).

EVALUATE: The total momentum can be calculated either as the vector sum of the momenta of the individual objects in the system, or as the total mass of the system times the velocity of the center of mass.

8.51 **IDENTIFY:** Use Eq. 8.28 to find the *x* and *y* coordinates of the center of mass of the machine part for each configuration of the part. In calculating the center of mass of the machine part, each uniform bar can be represented by a point mass at its geometrical center.

SET UP: Use coordinates with the axis at the hinge and the +*x* and +*y* axes along the horizontal and vertical bars in the figure in the problem. Let $(x_{\text{i}}, y_{\text{i}})$ and $(x_{\text{f}}, y_{\text{f}})$ be the coordinates of the bar before and after the vertical bar is pivoted. Let object 1 be the horizontal bar, object 2 be the vertical bar and 3 be the ball.

EXECUTE: $x_{\text{i}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(4.00\text{ kg})(0.750\text{ m}) + 0 + 0}{4.00\text{ kg} + 3.00\text{ kg} + 2.00\text{ kg}} = 0.333\text{ m}$.

$$y_{\text{i}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (3.00\text{ kg})(0.900\text{ m}) + (2.00\text{ kg})(1.80\text{ m})}{9.00\text{ kg}} = 0.700\text{ m}.$$

$$x_{\text{f}} = \frac{(4.00\text{ kg})(0.750\text{ m}) + (3.00\text{ kg})(-0.900\text{ m}) + (2.00\text{ kg})(-1.80\text{ m})}{9.00\text{ kg}} = -0.366\text{ m}.$$

$y_{\text{f}} = 0$. $x_{\text{f}} - x_{\text{i}} = -0.700\text{ m}$ and $y_{\text{f}} - y_{\text{i}} = -0.700\text{ m}$. The center of mass moves 0.700 m to the right and 0.700 m upward.

EVALUATE: The vertical bar moves upward and to the right so it is sensible for the center of mass of the machine part to move in these directions.

8.99 **IDENTIFY:** In Eq. 8.28 treat each straight piece as an object in the system.

SET UP: The center of mass of each piece of length *L* is at its center.

EXECUTE: **(a)** From symmetry, the center of mass is on the vertical axis, a distance $(L/2)\cos(\alpha/2)$ below the apex.

(b) The center of mass is on the vertical axis of symmetry, a distance $2(L/2)/3 = L/3$ above the center of the horizontal segment.

(c) Using the wire frame as a coordinate system, the coordinates of the center of mass are equal and each is equal to $(L/2)/2 = L/4$. The center of mass is along the bisector of the angle, a distance $L/\sqrt{8}$ from the corner.

(d) By symmetry, the center of mass is at the center of the equilateral triangle, a distance $(L/3)\sin 60^\circ = L/\sqrt{12}$ above the center of the horizontal segment.

EVALUATE: The center of mass need not lie on any point of the object, it can be in empty space.

8.100 **IDENTIFY:** There is no net horizontal external force so v_{cm} is constant.

SET UP: Let +*x* be to the right, with the origin at the initial position of the left-hand end of the canoe. $m_{\text{A}} = 45.0\text{ kg}$, $m_B = 60.0\text{ kg}$. The center of mass of the canoe is at its center.

EXECUTE: Initially, $v_{\text{cm}} = 0$, so the center of mass doesn't move. Initially, $x_{\text{cm1}} = \dfrac{m_A x_{A1} + m_B x_{B1}}{m_A + m_B}$. After she walks, $x_{\text{cm2}} = \dfrac{m_A x_{A2} + m_B x_{B2}}{m_A + m_B}$. $x_{\text{cm1}} = x_{\text{cm2}}$ gives $m_A x_{A1} + m_B x_{B1} = m_A x_{A2} + m_B x_{B2}$. She walks to a point 1.00 m from the right-hand end of the canoe, so she is 1.50 m to the right of the center of mass of the canoe and $x_{A2} = x_{B2} + 1.50\text{ m}$.

$(45.0 \text{ kg})(1.00 \text{ m}) + (60.0 \text{ kg})(2.50 \text{ m}) = (45.0 \text{ kg})(x_{B2} + 1.50 \text{ m}) + (60.0 \text{ kg})x_{B2}$.

$(105.0 \text{ kg})x_{B2} = 127.5 \text{ kg} \cdot \text{m}$ and $x_{B2} = 1.21 \text{ m}$. $x_{B2} - x_{B1} = 1.21 \text{ m} - 2.50 \text{ m} = -1.29 \text{ m}$. The canoe moves 1.29 m to the left.

EVALUATE: When the woman walks to the right, the canoe moves to the left. The woman walks 3.00 m to the right relative to the canoe and the canoe moves 1.29 m to the left, so she moves $3.00 \text{ m} - 1.29 \text{ m} = 1.71 \text{ m}$ to the right relative to the water. Note that this distance is $(60.0 \text{ kg}/45.0 \text{ kg})(1.29 \text{ m})$.

8.113 **IDENTIFY** and **SET UP:** $dm = \rho dV$. $dV = A dx$. Since the thin rod lies along the x axis, $y_{\text{cm}} = 0$. The mass of the rod is given by $M = \int dm$.

EXECUTE: **(a)** $x_{\text{cm}} = \dfrac{1}{M}\int_0^L x dm = \dfrac{\rho}{M} A \int_0^L x dx = \dfrac{\rho A}{M}\dfrac{L^2}{2}$. The volume of the rod is AL and $M = \rho AL$.

$x_{\text{cm}} = \dfrac{\rho A L^2}{2\rho AL} = \dfrac{L}{2}$. The center of mass of the uniform rod is at its geometrical center, midway between its ends.

(b) $x_{\text{cm}} = \dfrac{1}{M}\int_0^L x dm = \dfrac{1}{M}\int_0^L x\rho A dx = \dfrac{A\alpha}{M}\int_0^L x^2 dx = \dfrac{A\alpha L^3}{3M}$. $M = \int dm = \int_0^L \rho A dx = \alpha A \int_0^L x dx = \dfrac{\alpha A L^2}{2}$. Therefore,

$x_{\text{cm}} = \left(\dfrac{A\alpha L^3}{3}\right)\left(\dfrac{2}{\alpha A L^2}\right) = \dfrac{2L}{3}$.

EVALUATE: When the density increases with x, the center of mass is to the right of the center of the rod.