**9.5** **IDENTIFY** and **SET UP:** Use Eq.(9.3) to calculate the angular velocity and Eq.(9.2) to calculate the average angular velocity for the specified time interval.

**EXECUTE:** $\theta = \gamma t + \beta t^3$; $\gamma = 0.400$ rad/s, $\beta = 0.0120$ rad/s$^3$

**(a)** $\omega_z = \dfrac{d\theta}{dt} = \gamma + 3\beta t^2$

**(b)** At $t = 0$, $\omega_z = \gamma = 0.400$ rad/s

**(c)** At $t = 5.00$ s, $\omega_z = 0.400 \text{ rad/s} + 3(0.0120 \text{ rad/s}^3)(5.00 \text{ s})^2 = 1.30 \text{ rad/s}$

$$\omega_{\text{av-}z} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

For $t_1 = 0$, $\theta_1 = 0$.

For $\theta_2 = 5.00$ s, $\theta_2 = (0.400 \text{ rad/s})(5.00 \text{ s}) + (0.012 \text{ rad/s}^3)(5.00 \text{ s})^3 = 3.50 \text{ rad}$

So $\omega_{\text{av-}z} = \dfrac{3.50 \text{ rad} - 0}{5.00 \text{ s} - 0} = 0.700 \text{ rad/s}$.

**EVALUATE:** The average of the instantaneous angular velocities at the beginning and end of the time interval is $\frac{1}{2}(0.400 \text{ rad/s} + 1.30 \text{ rad/s}) = 0.850 \text{ rad/s}$. This is larger than $\omega_{\text{av-}z}$, because $\omega_z(t)$ is increasing faster than linearly.

**9.10** **IDENTIFY:** Apply the constant angular acceleration equations to the motion of the fan.

**(a) SET UP:** $\omega_{0z} = (500 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 8.333 \text{ rev/s}$, $\omega_z = (200 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 3.333 \text{ rev/s}$,

$t = 4.00$ s, $\alpha_z = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

**EXECUTE:** $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{3.333 \text{ rev/s} - 8.333 \text{ rev/s}}{4.00 \text{ s}} = -1.25 \text{ rev/s}^2$

$\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z} t + \frac{1}{2}\alpha_z t^2 = (8.333 \text{ rev/s})(4.00 \text{ s}) + \frac{1}{2}(-1.25 \text{ rev/s}^2)(4.00 \text{ s})^2 = 23.3 \text{ rev}$

**(b) SET UP:** $\omega_z = 0$ (comes to rest); $\omega_{0z} = 3.333$ rev/s; $\alpha_z = -1.25$ rev/s$^2$;

$t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

**EXECUTE:** $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{0 - 3.333 \text{ rev/s}}{-1.25 \text{ rev/s}^2} = 2.67 \text{ s}$

**EVALUATE:** The angular acceleration is negative because the angular velocity is decreasing. The average angular velocity during the 4.00 s time interval is 350 rev/min and $\theta - \theta_0 = \omega_{\text{av-}z} t$ gives $\theta - \theta_0 = 23.3$ rev, which checks.

**9.15** **IDENTIFY:** Apply constant angular acceleration equations.

**SET UP:** Let the direction the flywheel is rotating be positive.

$\theta - \theta_0 = 200$ rev, $\omega_{0z} = 500$ rev/min $= 8.333$ rev/s, $t = 30.0$ s.

**EXECUTE:** **(a)** $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = 5.00 \text{ rev/s} = 300 \text{ rpm}$

**(b)** Use the information in part (a) to find $\alpha_z$: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.1111 \text{ rev/s}^2$. Then $\omega_z = 0$, $\alpha_z = -0.1111 \text{ rev/s}^2$, $\omega_{0z} = 8.333$ rev/s in $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = 75.0$ s and $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives

$\theta - \theta_0 = 312$ rev.

**EVALUATE:** The mass and diameter of the flywheel are not used in the calculation.

**9.20** **IDENTIFY:** The linear distance the elevator travels, its speed and the magnitude of its acceleration are equal to the tangential displacement, speed and acceleration of a point on the rim of the disk. $s = r\theta$, $v = r\omega$ and $a = r\alpha$. In these equations the angular quantities must be in radians.

**SET UP:** 1 rev $= 2\pi$ rad. 1 rpm $= 0.1047$ rad/s. $\pi$ rad $= 180°$. For the disk, $r = 1.25$ m.

**EXECUTE:** **(a)** $v = 0.250$ m/s so $\omega = \dfrac{v}{r} = \dfrac{0.250 \text{ m/s}}{1.25 \text{ m}} = 0.200 \text{ rad/s} = 1.91 \text{ rpm}$.

**(b)** $a = \frac{1}{8}g = 1.225\ \text{m/s}^2$. $\alpha = \dfrac{a}{r} = \dfrac{1.225\ \text{m/s}^2}{1.25\ \text{m}} = 0.980\ \text{rad/s}^2$.

**(c)** $s = 3.25$ m. $\theta = \dfrac{s}{r} = \dfrac{3.25\ \text{m}}{1.25\ \text{m}} = 2.60\ \text{rad} = 149°$.

**EVALUATE:** When we use $s = r\theta$, $v = r\omega$ and $a_{\text{tan}} = r\alpha$ to solve for $\theta$, $\omega$ and $\alpha$, the results are in rad, rad/s and $\text{rad/s}^2$.

**9.25** **IDENTIFY** and **SET UP:** Use constant acceleration equations to find $\omega$ and $\alpha$ after each displacement. The use Eqs.(9.14) and (9.15) to find the components of the linear acceleration.

**EXECUTE:** **(a)** <u>at the start</u> $t = 0$

flywheel starts from rest so $\omega_z = \omega_{0z} = 0$

$a_{\text{tan}} = r\alpha = (0.300\ \text{m})(0.600\ \text{rad/s}^2) = 0.180\ \text{m/s}^2$

$a_{\text{rad}} = r\omega^2 = 0$

$a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = 0.180\ \text{m/s}^2$

**(b)** <u>$\theta - \theta_0 = 60°$</u>

$a_{\text{tan}} = r\alpha = 0.180\ \text{m/s}^2$

Calculate $\omega$:

$\theta - \theta_0 = 60°(\pi\ \text{rad}/180°) = 1.047\ \text{rad};\ \ \omega_{0z} = 0;\ \ \alpha_z = 0.600\ \text{rad/s}^2;\ \ \omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600\ \text{rad/s}^2)(1.047\ \text{rad})} = 1.121\ \text{rad/s}$ and $\omega = \omega_z$.

Then $a_{\text{rad}} = r\omega^2 = (0.300\ \text{m})(1.121\ \text{rad/s})^2 = 0.377\ \text{m/s}^2$.

$a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = \sqrt{(0.377\ \text{m/s}^2)^2 + (0.180\ \text{m/s}^2)^2} = 0.418\ \text{m/s}^2$

**(c)** <u>$\theta - \theta_0 = 120°$</u>

$a_{\text{tan}} = r\alpha = 0.180\ \text{m/s}^2$

Calculate $\omega$:

$\theta - \theta_0 = 120°(\pi\ \text{rad}/180°) = 2.094\ \text{rad};\ \ \omega_{0z} = 0;\ \ \alpha_z = 0.600\ \text{rad/s}^2;\ \ \omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600\ \text{rad/s}^2)(2.094\ \text{rad})} = 1.585\ \text{rad/s}$ and $\omega = \omega_z$.

Then $a_{\text{rad}} = r\omega^2 = (0.300\ \text{m})(1.585\ \text{rad/s})^2 = 0.754\ \text{m/s}^2$.

$a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = \sqrt{(0.754\ \text{m/s}^2)^2 + (0.180\ \text{m/s}^2)^2} = 0.775\ \text{m/s}^2$

**EVALUATE:** $\alpha$ is constant so $\alpha_{\text{tan}}$ is constant. $\omega$ increases so $a_{\text{rad}}$ increases.

**9.27** **IDENTIFY:** Use Eq.(9.15) and solve for *r*.

**SET UP:** $a_{\text{rad}} = r\omega^2$ so $r = a_{\text{rad}}/\omega^2$, where $\omega$ must be in rad/s

**EXECUTE:** $a_{\text{rad}} = 3000g = 3000(9.80\ \text{m/s}^2) = 29{,}400\ \text{m/s}^2$

$\omega = (5000\ \text{rev/min})\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 523.6\ \text{rad/s}$

Then $r = \dfrac{a_{\text{rad}}}{\omega^2} = \dfrac{29{,}400\ \text{m/s}^2}{(523.6\ \text{rad/s})^2} = 0.107\ \text{m}.$

**EVALUATE:** The diameter is then 0.214 m, which is larger than 0.127 m, so the claim is *not* realistic.

**9.67** **IDENTIFY:** The angular acceleration $\alpha$ of the disk is related to the linear acceleration *a* of the ball by $a = R\alpha$. Since the acceleration is not constant, use $\omega_z - \omega_{0z} = \int_0^t \alpha_z dt$ and $\theta - \theta_0 = \int_0^t \omega_z dt$ to relate $\theta$, $\omega_z$, $\alpha_z$ and *t* for the disk. $\omega_{0z} = 0$.

**SET UP:** $\int t^n dt = \dfrac{1}{n+1} t^{n+1}$. In $a = R\alpha$, $\alpha$ is in $\text{rad/s}^2$.

**EXECUTE:** **(a)** $A = \dfrac{a}{t} = \dfrac{1.80\ \text{m/s}^2}{3.00\ \text{s}} = 0.600\ \text{m/s}^3$

**(b)** $\alpha = \dfrac{a}{R} = \dfrac{(0.600\ \text{m/s}^3)t}{0.250\ \text{m}} = (2.40\ \text{rad/s}^3)t$

**(c)** $\omega_z = \int_0^t (2.40\ \text{rad/s}^3)t dt = (1.20\ \text{rad/s}^3)t^2$ . $\omega_z = 15.0$ rad/s for $t = \sqrt{\dfrac{15.0\ \text{rad/s}}{1.20\ \text{rad/s}^3}} = 3.54\ \text{s}$ .

**(d)** $\theta - \theta_0 = \int_0^t \omega_z dt = \int_0^t (1.20\ \text{rad/s}^3)t^2 dt = (0.400\ \text{rad/s}^3)t^3$ . For $t = 3.54\ \text{s}$ , $\theta - \theta_0 = 17.7\ \text{rad}$ .

**EVALUATE:** If the disk had turned at a constant angular velocity of 15.0 rad/s for 3.54 s it would have turned through an angle of 53.1 rad in 3.54 s. It actually turns through less than half this because the angular velocity is increasing in time and is less than 15.0 rad/s at all but the end of the interval.