**3.13.** **IDENTIFY:** The car moves in projectile motion. The car travels $21.3\text{ m} - 1.80\text{ m} = 19.5\text{ m}$ downward during the time it travels 61.0 m horizontally.

**SET UP:** Take $+y$ to be downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

**EXECUTE:** Use the vertical motion to find the time in the air:

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 \text{ gives } t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(19.5\text{ m})}{9.80\text{ m/s}^2}} = 1.995\text{ s}$$

Then $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{61.0\text{ m}}{1.995\text{ s}} = 30.6\text{ m/s}$.

**(b)** $v_x = 30.6\text{m/s}$ since $a_x = 0$. $v_y = v_{0y} + a_y t = -19.6\text{m/s}$. $v = \sqrt{v_x^2 + v_y^2} = 36.3\text{m/s}$.

**EVALUATE:** We calculate the final velocity by calculating its *x* and *y* components.

**3.14.** **IDENTIFY:** The marble moves with projectile motion, with initial velocity that is horizontal and has magnitude $v_0$. Treat the horizontal and vertical motions separately. If $v_0$ is too small the marble will land to the left of the hole and if $v_0$ is too large the marble will land to the right of the hole.

**SET UP:** Let $+x$ be horizontal to the right and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$

**EXECUTE:** Use the vertical motion to find the time it takes the marble to reach the height of the level ground; $y - y_0 = -2.75\text{ m}$. $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(-2.75\text{ m})}{-9.80\text{ m/s}^2}} = 0.749\text{ s}$. The time does not depend on $v_0$.

Minimum $v_0$: $x - x_0 = 2.00\text{ m}$, $t = 0.749\text{ s}$. $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$ gives $v_0 = \dfrac{x - x_0}{t} = \dfrac{2.00\text{ m}}{0.749\text{ s}} = 2.67\text{ m/s}$.

Maximum $v_0$: $x - x_0 = 3.50\text{ m}$ and $v_0 = \dfrac{3.50\text{ m}}{0.749\text{ s}} = 4.67\text{ m/s}$.

**EVALUATE:** The horizontal and vertical motions are independent and are treated separately. Their only connection is that the time is the same for both.

**3.15.** **IDENTIFY:** The ball moves with projectile motion with an initial velocity that is horizontal and has magnitude $v_0$. The height *h* of the table and $v_0$ are the same; the acceleration due to gravity changes from $g_E = 9.80\text{ m/s}^2$ on earth to $g_X$ on planet X.

**SET UP:** Let $+x$ be horizontal and in the direction of the initial velocity of the marble and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -g$, where *g* is either $g_E$ or $g_X$.

**EXECUTE:** Use the vertical motion to find the time in the air: $y - y_0 = -h$. $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2h}{g}}$.

Then $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$ gives $x - x_0 = v_{0x}t = v_0\sqrt{\dfrac{2h}{g}}$. $x - x_0 = D$ on earth and $2.76D$ on Planet X.

$(x - x_0)\sqrt{g} = v_0\sqrt{2h}$, which is constant, so $D\sqrt{g_E} = 2.76D\sqrt{g_X}$. $g_X = \dfrac{g_E}{(2.76)^2} = 0.131 g_E = 1.28\text{ m/s}^2$.

**EVALUATE:** On Planet X the acceleration due to gravity is less, it takes the ball longer to reach the floor, and it travels farther horizontally.

**3.19.** **IDENTIFY:** The baseball moves in projectile motion. In part (c) first calculate the components of the velocity at this point and then get the resultant velocity from its components.

**SET UP:** First find the *x*- and *y*-components of the initial velocity. Use coordinates where the $+y$-direction is upward, the $+x$-direction is to the right and the origin is at the point where the baseball leaves the bat.

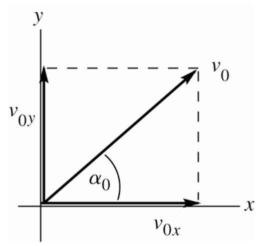


**Figure 3.19a**

$$v_{0x} = v_0 \cos\alpha_0 = (30.0\text{ m/s})\cos 36.9° = 24.0\text{ m/s}$$
$$v_{0y} = v_0 \sin\alpha_0 = (30.0\text{ m/s})\sin 36.9° = 18.0\text{ m/s}$$

Use constant acceleration equations for the *x* and *y* motions, with $a_x = 0$ and $a_y = -g$.

**EXECUTE:** **(a)** y-component (vertical motion):

$y - y_0 = +10.0\text{ m/s},\;\; v_{0y} = 18.0\text{ m/s},\;\; a_y = -9.80\text{ m/s}^2,\;\; t = ?$

$y - y_0 = v_{0y} + \frac{1}{2}a_y t^2$

$10.0\text{ m} = (18.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$

$(4.90\text{ m/s}^2)t^2 - (18.0\text{ m/s})t + 10.0\text{ m} = 0$

Apply the quadratic formula: $t = \frac{1}{9.80}\left[18.0 \pm \sqrt{(-18.0)^2 - 4(4.90)(10.0)}\right]\text{ s} = (1.837 \pm 1.154)\text{ s}$

The ball is at a height of 10.0 above the point where it left the bat at $t_1 = 0.683\text{ s}$ and at $t_2 = 2.99\text{ s}$. At the earlier time the ball passes through a height of 10.0 m as its way up and at the later time it passes through 10.0 m on its way down.

**(b)** $v_x = v_{0x} = +24.0\text{ m/s}$, at all times since $a_x = 0$.

$v_y = v_{0y} + a_y t$

$t_1 = 0.683\text{ s}$: $v_y = +18.0\text{ m/s} + (-9.80\text{ m/s}^2)(0.683\text{ s}) = +11.3\text{ m/s}$. ($v_y$ is positive means that the ball is traveling upward at this point.

$t_2 = 2.99\text{ s}$: $v_y = +18.0\text{ m/s} + (-9.80\text{ m/s}^2)(2.99\text{ s}) = -11.3\text{ m/s}$. ($v_y$ is negative means that the ball is traveling downward at this point.)

**(c)** $v_x = v_{0x} = 24.0\text{ m/s}$

Solve for $v_y$:

$v_y = ?,\;\; y - y_0 = 0$ (when ball returns to height where motion started),

$a_y = -9.80\text{ m/s}^2,\;\; v_{0y} = +18.0\text{ m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -v_{0y} = -18.0\text{ m/s}$ (negative, since the baseball must be traveling downward at this point)

Now that have the components can solve for the magnitude and direction of $\vec{v}$.

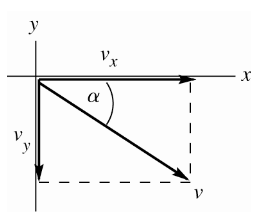


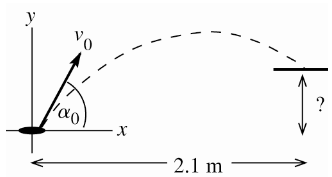
**Figure 3.19b**

$$v = \sqrt{v_x^2 + v_y^2}$$
$$v = \sqrt{(24.0\text{ m/s})^2 + (-18.0\text{ m/s})^2} = 30.0\text{ m/s}$$
$$\tan\alpha = \frac{v_y}{v_x} = \frac{-18.0\text{ m/s}}{24.0\text{ m/s}}$$
$$\alpha = -36.9°,\;\; 36.9° \text{ below the horizontal}$$

The velocity of the ball when it returns to the level where it left the bat has magnitude 30.0 m/s and is directed at an angle of $36.9°$ below the horizontal.

**EVALUATE:** The discussion in parts (a) and (b) explains the significance of two values of *t* for which $y - y_0 = +10.0\text{ m}$. When the ball returns to its initial height, our results give that its speed is the same as its initial speed and the angle of its velocity below the horizontal is equal to the angle of its initial velocity above the horizontal; both of these are general results.

**3.21.** **IDENTIFY:** Take the origin of coordinates at the point where the quarter leaves your hand and take positive *y* to be upward. The quarter moves in projectile motion, with $a_x = 0$, and $a_y = -g$. It travels vertically for the time it takes it to travel horizontally 2.1 m.




$v_{0x} = v_0 \cos\alpha_0 = (6.4 \text{ m/s})\cos 60°$

$v_{0x} = 3.20 \text{ m/s}$

$v_{0y} = v_0 \sin\alpha_0 = (6.4 \text{ m/s})\sin 60°$

$v_{0y} = 5.54 \text{ m/s}$

**Figure 3.21**

**(a) SET UP:** Use the horizontal (*x*-component) of motion to solve for *t*, the time the quarter travels through the air:

$t = ?,\ \ x - x_0 = 2.1 \text{ m},\ \ v_{0x} = 3.2 \text{ m/s},\ \ a_x = 0$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = v_{0x}t,$ since $a_x = 0$

**EXECUTE:** $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{2.1 \text{ m}}{3.2 \text{ m/s}} = 0.656 \text{ s}$

**SET UP:** Now find the vertical displacement of the quarter after this time:

$y - y_0 = ?,\ \ a_y = -9.80 \text{ m/s}^2,\ \ v_{0y} = +5.54 \text{ m/s},\ \ t = 0.656 \text{ s}$

$y - y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

**EXECUTE:** $y - y_0 = (5.54 \text{ m/s})(0.656 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.656 \text{ s})^2 = 3.63 \text{ m} - 2.11 \text{ m} = 1.5 \text{ m}.$

**(b) SET UP:** $v_y = ?,\ \ t = 0.656 \text{ s},\ \ a_y = -9.80 \text{ m/s}^2,\ \ v_{0y} = +5.54 \text{ m/s}$

$v_y = v_{0y} + a_y t$

**EXECUTE:** $v_y = 5.54 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.656 \text{ s}) = -0.89 \text{ m/s}.$

**EVALUATE:** The minus sign for $v_y$ indicates that the *y*-component of $\vec{v}$ is downward. At this point the quarter has passed through the highest point in its path and is on its way down. The horizontal range if it returned to its original height (it doesn't!) would be 3.6 m. It reaches its maximum height after traveling horizontally 1.8 m, so at $x - x_0 = 2.1 \text{ m}$ it is on its way down.

**3.32.** **IDENTIFY:** Each planet moves in a circular orbit and therefore has acceleration $a_{\text{rad}} = v^2/R$.

**SET UP:** The radius of the earth's orbit is $r = 1.50 \times 10^{11} \text{ m}$ and its orbital period is $T = 365 \text{ days} = 3.16 \times 10^7 \text{ s}$. For Mercury, $r = 5.79 \times 10^{10} \text{ m}$ and $T = 88.0 \text{ days} = 7.60 \times 10^6 \text{ s}$.

**EXECUTE:** **(a)** $v = \dfrac{2\pi r}{T} = 2.98 \times 10^4 \text{ m/s}$

**(b)** $a_{\text{rad}} = \dfrac{v^2}{r} = 5.91 \times 10^{-3} \text{ m/s}^2$.

**(c)** $v = 4.79 \times 10^4 \text{ m/s}$, and $a_{\text{rad}} = 3.96 \times 10^{-2} \text{ m/s}^2$.

**EVALUATE:** Mercury has a larger orbital velocity and a larger radial acceleration than earth.

**3.33.** **IDENTIFY:** Uniform circular motion.

**SET UP:** Since the magnitude of $\vec{v}$ is constant. $v_{\tan} = \dfrac{d|\vec{v}|}{dt} = 0$ and the resultant acceleration is equal to the radial component. At each point in the motion the radial component of the acceleration is directed in toward the center of the circular path and its magnitude is given by $v^2/R$.

**EXECUTE:** **(a)** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(7.00 \text{ m/s})^2}{14.0 \text{ m}} = 3.50 \text{ m/s}^2,$ upward.

**(b)** The radial acceleration has the same magnitude as in part (a), but now the direction toward the center of the circle is downward. The acceleration at this point in the motion is $3.50 \text{ m/s}^2$, downward.

**(c) SET UP:** The time to make one rotation is the period *T*, and the speed *v* is the distance for one revolution divided by *T*.

**EXECUTE:** $v = \dfrac{2\pi R}{T}$ so $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(14.0 \text{ m})}{7.00 \text{ m/s}} = 12.6 \text{ s}$

**EVALUATE:** The radial acceleration is constant in magnitude since $v$ is constant and is at every point in the motion directed toward the center of the circular path. The acceleration is perpendicular to $\vec{v}$ and is nonzero because the direction of $\vec{v}$ changes.

**3.38.** **IDENTIFY:** Calculate the rower's speed relative to the shore for each segment of the round trip.
**SET UP:** The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream.
**EXECUTE:** The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min).

The total time the rower takes is $\frac{1.5 \text{ km}}{6.8 \text{ km/h}} + \frac{1.5 \text{ km}}{1.2 \text{ km/h}} = 1.47 \text{ h} = 88.2 \text{ min}.$

**EVALUATE:** It takes the rower longer, even though for half the distance his speed is greater than 4.0 km/h. The rower spends more time at the slower speed.

**3.40.** **IDENTIFY:** Use the relation that relates the relative velocities.
**SET UP:** The relative velocities are the velocity of the plane relative to the ground, $\vec{v}_{\text{P/G}}$, the velocity of the plane relative to the air, $\vec{v}_{\text{P/A}}$, and the velocity of the air relative to the ground, $\vec{v}_{\text{A/G}}$. $\vec{v}_{\text{P/G}}$ must due west and $\vec{v}_{\text{A/G}}$ must be south. $v_{\text{A/G}} = 80 \text{ km/h}$ and $v_{\text{P/A}} = 320 \text{ km/h}$. $\vec{v}_{\text{P/G}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/G}}$. The relative velocity addition diagram is given in Figure 3.40.

**EXECUTE:** **(a)** $\sin\theta = \frac{v_{\text{A/G}}}{v_{\text{P/A}}} = \frac{80 \text{ km/h}}{320 \text{ km/h}}$ and $\theta = 14°$, north of west.

**(b)** $v_{\text{P/G}} = \sqrt{v_{\text{P/A}}^2 - v_{\text{A/G}}^2} = \sqrt{(320 \text{ km/h})^2 - (80.0 \text{ km/h})^2} = 310 \text{ km/h}$.

**EVALUATE:** To travel due west the velocity of the plane relative to the air must have a westward component and also a component that is northward, opposite to the wind direction.

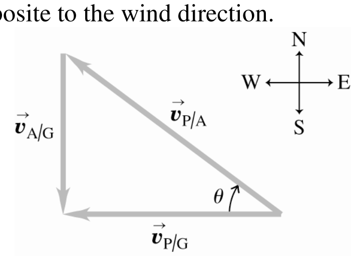



**Figure 3.40**

**3.77.** **IDENTIFY:** $v_x = dx/dt$, $v_y = dy/dt$, $a_x = dv_x/dt$ and $a_y = dv_y/dt$.

**SET UP:** $\frac{d(\sin\omega t)}{dt} = \omega\cos(\omega t)$ and $\frac{d(\cos\omega t)}{dt} = -\omega\sin(\omega t)$.

**EXECUTE:** **(a)** The path is sketched in Figure 3.77.

**(b)** To find the velocity components, take the derivative of $x$ and $y$ with respect to time: $v_x = R\omega(1-\cos\omega t)$, and $v_y = R\omega\sin\omega t$. To find the acceleration components, take the derivative of $v_x$ and $v_y$ with respect to time: $a_x = R\omega^2 \sin\omega t$, and $a_y = R\omega^2 \cos\omega t$.

**(c)** The particle is at rest $(v_y = v_x = 0)$ every period, namely at $t = 0,\ 2\pi/\omega,\ 4\pi/\omega, \ldots$ At that time, $x = 0,\ 2\pi R,\ 4\pi R, \ldots$; and $y = 0$. The acceleration is $a = R\omega^2$ in the $+y$-direction.

**(d)** No, since $a = \left[\left(R\omega^2 \sin\omega t\right)^2 + \left(R\omega^2 \cos\omega t\right)^2\right]^{1/2} = R\omega^2$. The magnitude of the acceleration is the same as for uniform circular motion.

**EVALUATE:** The velocity is tangent to the path. $v_{0x}$ is always positive; $v_y$ changes sign during the motion.

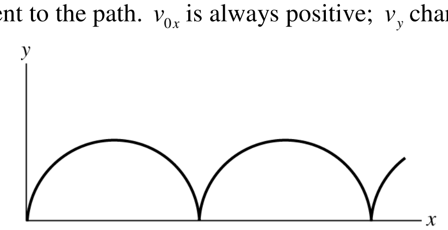



**Figure 3.77**

**3.79.** **IDENTIFY:** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{4\pi^2 R}{T^2}$. All points on the centrifuge have the same period *T*.

**SET UP:** The period *T* in seconds is related to *n*, the number of revolutions per minute, by $n = \dfrac{60\text{ s/min}}{T}$.

**EXECUTE:** **(a)** $\dfrac{a_{\text{rad}}}{R} = \dfrac{4\pi^2}{T^2}$, which is constant. $\dfrac{a_{\text{rad},1}}{R_1} = \dfrac{a_{\text{rad},2}}{R_2}$. Let $R_1 = R$, so $a_{\text{rad},1} = 5.00g$ and let $R_2 = R/2$.

$$a_{\text{rad},2} = a_{\text{rad},1}\left(\frac{R_2}{R_1}\right) = (5.00g)(1/2) = 2.50g\ .$$

**(b)** $T = \left(\dfrac{60\text{ s/min}}{n}\right)$ and $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ gives $a_{\text{rad}} = 4\pi^2 R n^2/(60\text{ s/min})^2$. $\dfrac{a_{\text{rad}}}{n^2} = \dfrac{4\pi^2 R}{(60\text{ s/min})^2}$, which is constant.

$\dfrac{a_{\text{rad},1}}{n_1^2} = \dfrac{a_{\text{rad},2}}{n_2^2}$. Let $a_{\text{rad},1} = 5.00g$, so $n_1 = n$ and $a_{\text{rad},2} = 5g_{\text{Mercury}} = 5(0.378)g$. Then

$$n_2 = n_1\sqrt{\frac{a_{\text{rad},2}}{a_{\text{rad},1}}} = n\sqrt{\frac{5(0.378)g}{5.00g}} = 0.615n\ .$$

**EVALUATE:** The radial acceleration is less for points closer to the rotation axis. Since $g_{\text{Mercury}} < g$, a smaller rotation rate is required to produce $5g_{\text{Mercury}}$ than to produce $5g$.

**3.86.** **IDENTIFY:** **(a)** The ball moves in projectile motion. When it is moving horizontally, $v_y = 0$.

**SET UP:** Let $+x$ be to the right and let $+y$ be upward. $a_x = 0$, $a_y = -g$.

**EXECUTE:** **(a)** $v_{0y} = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(4.90\text{ m})} = 9.80\text{ m/s}$.

**(b)** $v_{0y}/g = 1.00\text{ s}$.

**(c)** The horizontal component of the velocity of the ball relative to the man is $\sqrt{(10.8\text{ m/s})^2 - (9.80\text{ m/s})^2} = 4.54\text{ m/s}$, the horizontal component of the velocity relative to the hoop is $4.54\text{ m/s} + 9.10\text{ m/s} = 13.6\text{ m/s}$, and the man must be 13.6 m in front of the hoop at release.

**(d)** Relative to the flat car, the ball is projected at an angle $\theta = \tan^{-1}\left(\dfrac{9.80\text{ m/s}}{4.54\text{ m/s}}\right) = 65°$. Relative to the ground the angle is $\theta = \tan^{-1}\left(\dfrac{9.80\text{ m/s}}{4.54\text{ m/s} + 9.10\text{ m/s}}\right) = 35.7°$.

**EVALUATE:** In both frames of reference the ball moves in a parabolic path with $a_x = 0$ and $a_y = -g$. The only difference between the description of the motion in the two frames is the horizontal component of the ball's velocity.