

5.66. IDENTIFY: The system is in equilibrium. Apply Newton's 1st law to block A, to the hanging weight and to the knot where the cords meet. Target variables are the two forces.

(a) SET UP: The free-body diagram for the hanging block is given in Figure 5.66a.

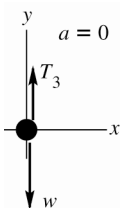


Figure 5.66a

EXECUTE:

$$\sum F_y = ma_y$$

$$T_3 - w = 0$$

$$T_3 = 12.0 \text{ N}$$

SET UP: The free-body diagram for the knot is given in Figure 5.66b.

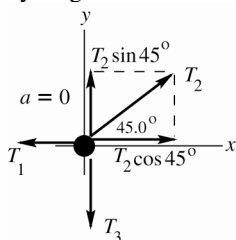


Figure 5.66b

EXECUTE:

$$\sum F_y = ma_y$$

$$T_2 \sin 45.0^\circ - T_3 = 0$$

$$T_2 = \frac{T_3}{\sin 45.0^\circ} = \frac{12.0 \text{ N}}{\sin 45.0^\circ}$$

$$T_2 = 17.0 \text{ N}$$

$$\sum F_x = ma_x$$

$$T_2 \cos 45.0^\circ - T_1 = 0$$

$$T_1 = T_2 \cos 45.0^\circ = 12.0 \text{ N}$$

SET UP: The free-body diagram for block A is given in Figure 5.66c.

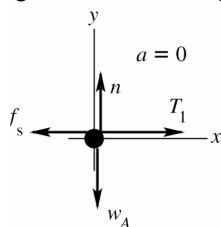


Figure 5.66c

EXECUTE:

$$\sum F_x = ma_x$$

$$T_1 - f_s = 0$$

$$f_s = T_1 = 12.0 \text{ N}$$

EVALUATE: Also can apply $\sum F_y = ma_y$ to this block:

$$n - w_A = 0$$

$$n = w_A = 60.0 \text{ N}$$

Then $\mu_s n = (0.25)(60.0 \text{ N}) = 15.0 \text{ N}$; this is the maximum possible value for the static friction force. We see that $f_s < \mu_s n$; for this value of w the static friction force can hold the blocks in place.

(b) SET UP: We have all the same free-body diagrams and force equations as in part (a) but now the static friction force has its largest possible value, $f_s = \mu_s n = 15.0 \text{ N}$. Then $T_1 = f_s = 15.0 \text{ N}$.

EXECUTE: From the equations for the forces on the knot

$$T_2 \cos 45.0^\circ - T_1 = 0 \text{ implies } T_2 = T_1 / \cos 45.0^\circ = \frac{15.0 \text{ N}}{\cos 45.0^\circ} = 21.2 \text{ N}$$

$$T_2 \sin 45.0^\circ - T_3 = 0 \text{ implies } T_3 = T_2 \sin 45.0^\circ = (21.2 \text{ N}) \sin 45.0^\circ = 15.0 \text{ N}$$

And finally $T_3 - w = 0$ implies $w = T_3 = 15.0 \text{ N}$.

EVALUATE: Compared to part (a), the friction is larger in part (b) by a factor of $(15.0/12.0)$ and w is larger by this same ratio.

5.73. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the package. Calculate a and then use a constant acceleration equation to describe the motion.

SET UP: Let $+x$ be directed up the ramp.

EXECUTE: (a) $F_{\text{net}} = -mg \sin 37^\circ - f_k = -mg \sin 37^\circ - \mu_k mg \cos 37^\circ = ma$ and

$$a = -(9.8 \text{ m/s}^2)(0.602 + (0.30)(0.799)) = -8.25 \text{ m/s}^2$$

Since we know the length of the slope, we can use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $x_0 = 0$ and $v_x = 0$ at the top.

$$v_0^2 = -2ax = -2(-8.25 \text{ m/s}^2)(8.0 \text{ m}) = 132 \text{ m}^2/\text{s}^2 \text{ and } v_0 = \sqrt{132 \text{ m}^2/\text{s}^2} = 11.5 \text{ m/s}$$

(b) For the trip back down the slope, gravity and the friction force operate in opposite directions to each other.

$$F_{\text{net}} = -mg \sin 37^\circ + \mu_k mg \cos 37^\circ = ma \text{ and}$$

$$a = g(-\sin 37^\circ + 0.30 \cos 37^\circ) = (9.8 \text{ m/s}^2)((-0.602) + (0.30)(0.799)) = -3.55 \text{ m/s}^2 .$$

Now we have $v_0 = 0$, $x_0 = -8.0 \text{ m}$, $x = 0$ and $v^2 = v_0^2 + 2a(x - x_0) = 0 + 2(-3.55 \text{ m/s}^2)(-8.0 \text{ m}) = 56.8 \text{ m}^2/\text{s}^2$, so

$$v = \sqrt{56.8 \text{ m}^2/\text{s}^2} = 7.54 \text{ m/s} .$$

EVALUATE: In both cases, moving up the incline and moving down the incline, the acceleration is directed down the incline. The magnitude of a is greater when the package is going up the incline, because $mg \sin 37^\circ$ and f_k are in the same direction whereas when the package is going down these two forces are in opposite directions.

5.83. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each block. Forces between the blocks are related by Newton's 3rd law. The target variable is the force F . Block B is pulled to the left at constant speed, so block A moves to the right at constant speed and $a = 0$ for each block.

SET UP: The free-body diagram for block A is given in Figure 5.83a. n_{BA} is the normal force that B exerts on A .

$f_{BA} = \mu_k n_{BA}$ is the kinetic friction force that B exerts on A . Block A moves to the right relative to B , and f_{BA} opposes this motion, so f_{BA} is to the left.

Note also that F acts just on B , not on A .

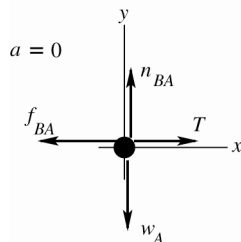


Figure 5.83a

EXECUTE:

$$\sum F_y = ma_y$$

$$n_{BA} - w_A = 0$$

$$n_{BA} = 1.40 \text{ N}$$

$$f_{BA} = \mu_k n_{BA} = (0.30)(1.40 \text{ N}) = 0.420 \text{ N}$$

$$\sum F_x = ma_x$$

$$T - f_{BA} = 0$$

$$T = f_{BA} = 0.420 \text{ N}$$

SET UP: The free-body diagram for block B is given in Figure 5.83b.

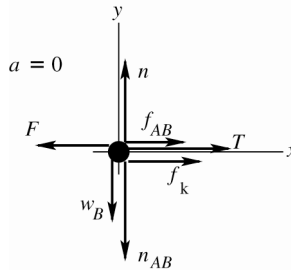


Figure 5.83b

EXECUTE: n_{AB} is the normal force that block A exerts on block B . By Newton's third law n_{AB} and n_{BA} are equal in magnitude and opposite in direction, so $n_{AB} = 1.40 \text{ N}$. f_{AB} is the kinetic friction force that A exerts on B . Block B moves to the left relative to A and f_{AB} opposes this motion, so f_{AB} is to the right.

$$f_{AB} = \mu_k n_{AB} = (0.30)(1.40 \text{ N}) = 0.420 \text{ N} .$$

n and f_k are the normal and friction force exerted by the floor on block B ; $f_k = \mu_k n$. Note that block B moves to the left relative to the floor and f_k opposes this motion, so f_k is to the right.

$$\sum F_y = ma_y$$

$$n - w_B - n_{AB} = 0$$

$$n = w_B + n_{AB} = 4.20 \text{ N} + 1.40 \text{ N} = 5.60 \text{ N}$$

$$\text{Then } f_k = \mu_k n = (0.30)(5.60 \text{ N}) = 1.68 \text{ N}.$$

$$\sum F_x = ma_x$$

$$f_{AB} + T + f_k - F = 0$$

$$F = T + f_{AB} + f_k = 0.420 \text{ N} + 0.420 \text{ N} + 1.68 \text{ N} = 2.52 \text{ N}$$

EVALUATE: Note that f_{AB} and f_{BA} are a third law action-reaction pair, so they must be equal in magnitude and opposite in direction and this is indeed what our calculation gives.

5.90. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the box.

SET UP: The box has an upward acceleration of $a = 1.90 \text{ m/s}^2$.

EXECUTE: The floor exerts an upward force n on the box, obtained from $n - mg = ma$, or $n = m(a + g)$. The friction force that needs to be balanced is

$$\mu_k n = \mu_k m(a + g) = (0.32)(28.0 \text{ kg})(1.90 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 105 \text{ N}.$$

EVALUATE: If the elevator wasn't accelerating the normal force would be $n = mg$ and the friction force that would have to be overcome would be 87.8 N. The upward acceleration increases the normal force and that increases the friction force.

5.92. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each block.

SET UP: Use coordinates where $+x$ is directed down the incline.

EXECUTE: (a) Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; *i.e.*, the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block, $(4.00 \text{ kg})g(\sin 30^\circ - (0.25)\cos 30^\circ) - T = (4.00 \text{ kg})a$, or $11.11 \text{ N} - T = (4.00 \text{ kg})a$, and similarly for the larger, $15.44 \text{ N} + T = (8.00 \text{ kg})a$. Adding these two relations, $26.55 \text{ N} = (12.00 \text{ kg})a$, $a = 2.21 \text{ m/s}^2$.

(b) Substitution into either of the above relations gives $T = 2.27 \text{ N}$.

(c) The string will be slack. The 4.00-kg block will have $a = 2.78 \text{ m/s}^2$ and the 8.00-kg block will have $a = 1.93 \text{ m/s}^2$, until the 4.00-kg block overtakes the 8.00-kg block and collides with it.

EVALUATE: If the string is cut the acceleration of each block will be independent of the mass of that block and will depend only on the slope angle and the coefficient of kinetic friction. The 8.00-kg block would have a smaller acceleration even though it has a larger mass, since it has a larger μ_k .

5.108. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the person and to the cart.

SET UP: The apparent weight, w_{app} , which is the same as the upward force on the person exerted by the car seat.

EXECUTE: (a) The apparent weight is the actual weight of the person minus the centripetal force needed to keep him moving in its circular path:

$$w_{\text{app}} = mg - \frac{mv^2}{R} = (70 \text{ kg}) \left[(9.8 \text{ m/s}^2) - \frac{(12 \text{ m/s})^2}{40 \text{ m}} \right] = 434 \text{ N}.$$

(b) The cart will lose contact with the surface when its apparent weight is zero; *i.e.*, when the road no longer has to exert any upward force on it: $mg - \frac{mv^2}{R} = 0$. $v = \sqrt{Rg} = \sqrt{(40 \text{ m})(9.8 \text{ m/s}^2)} = 19.8 \text{ m/s}$. The answer doesn't

depend on the cart's mass, because the centripetal force needed to hold it on the road is proportional to its mass and so to its weight, which provides the centripetal force in this situation.

EVALUATE: At the speed calculated in part (b), the downward force needed for circular motion is provided by gravity. For speeds greater than this more, downward force is needed and there is no source for it and the cart leaves the circular path. For speeds less than this, less downward force than gravity is needed, so the roadway must exert an upward vertical force.

5.109. (a) IDENTIFY: Use the information given about Jena to find the time t for one revolution of the merry-go-round. Her acceleration is a_{rad} , directed in toward the axis. Let \vec{F}_1 be the horizontal force that keeps her from sliding off.

Let her speed be v_1 and let R_1 be her distance from the axis. Apply $\sum \vec{F} = m\vec{a}$ to Jena, who moves in uniform circular motion.

SET UP: The free-body diagram for Jena is sketched in Figure 5.109a

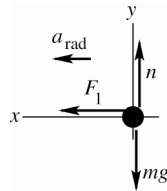


Figure 5.109a

EXECUTE:

$$\sum F_x = ma_x$$

$$F_1 = ma_{\text{rad}}$$

$$F_1 = m \frac{v_1^2}{R_1}, \quad v_1 = \sqrt{\frac{R_1 F_1}{m}} = 1.90 \text{ m/s}$$

The time for one revolution is $t = \frac{2\pi R_1}{v_1} = 2\pi R_1 \sqrt{\frac{m}{R_1 F_1}}$. Jackie goes around once in the same time but her speed

(v_2) and the radius of her circular path (R_2) are different.

$$v_2 = \frac{2\pi R_2}{t} = 2\pi R_2 \left(\frac{1}{2\pi R_1} \right) \sqrt{\frac{R_1 F_1}{m}} = \frac{R_2}{R_1} \sqrt{\frac{R_1 F_1}{m}}$$

IDENTIFY: Now apply $\sum \vec{F} = m\vec{a}$ to Jackie. She also moves in uniform circular motion.

SET UP: The free-body diagram for Jackie is sketched in Figure 5.109b.

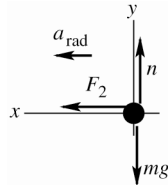


Figure 5.109b

EXECUTE:

$$\sum F_x = ma_x$$

$$F_2 = ma_{\text{rad}}$$

$$F_2 = m \frac{v_2^2}{R_2} = \left(\frac{m}{R_2} \right) \left(\frac{R_2^2}{R_1^2} \right) \left(\frac{R_1 F_1}{m} \right) = \left(\frac{R_2}{R_1} \right) F_1 = \left(\frac{3.60 \text{ m}}{1.80 \text{ m}} \right) (60.0 \text{ N}) = 120.0 \text{ N}$$

$$(b) \quad F_2 = m \frac{v_2^2}{R_2}, \text{ so } v_2 = \sqrt{\frac{F_2 R_2}{m}} = \sqrt{\frac{(120.0 \text{ N})(3.60 \text{ m})}{30.0 \text{ kg}}} = 3.79 \text{ m/s}$$

EVALUATE: Both girls rotate together so have the same period T . By Eq.(5.16), a_{rad} is larger for Jackie so the force on her is larger. Eq.(5.15) says $R_1/v_1 = R_2/v_2$ so $v_2 = v_1(R_2/R_1)$; this agrees with our result in (a).

5.110. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the passenger. The passenger has acceleration a_{rad} , directed inward toward the center of the circular path.

SET UP: The passenger's velocity is $v = 2\pi R/t = 8.80$ m/s. The vertical component of the seat's force must balance the passenger's weight and the horizontal component must provide the centripetal force.

EXECUTE: (a) $F_{\text{seat}} \sin \theta = mg = 833 \text{ N}$ and $F_{\text{seat}} \cos \theta = \frac{mv^2}{R} = 188 \text{ N}$. Therefore

$\tan \theta = (833 \text{ N})/(188 \text{ N}) = 4.43$; $\theta = 77.3^\circ$ above the horizontal. The magnitude of the net force exerted by the seat (note that this is not the net force on the passenger) is

$$F_{\text{seat}} = \sqrt{(833 \text{ N})^2 + (188 \text{ N})^2} = 854 \text{ N}$$

(b) The magnitude of the force is the same, but the horizontal component is reversed.

EVALUATE: At the highest point in the motion, $F_{\text{seat}} = mg - m \frac{v^2}{R} = 645 \text{ N}$. At the lowest point in the motion,

$$F_{\text{seat}} = mg + m \frac{v^2}{R} = 1021 \text{ N}. \text{ The result in parts (a) and (b) lies between these extreme values.}$$

5.115. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the circular motion of the bead. Also use Eq.(5.16) to relate a_{rad} to the period of rotation T .

SET UP: The bead and hoop are sketched in Figure 5.115a.

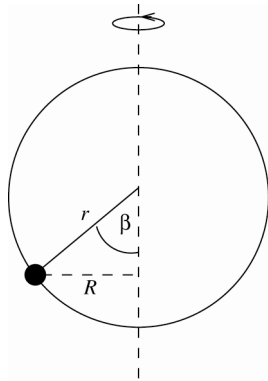


Figure 5.115a

The bead moves in a circle of radius
 $R = r \sin \beta$.
 The normal force exerted on the bead by
 the hoop is radially inward.

The free-body diagram for the bead is sketched in Figure 5.115b.

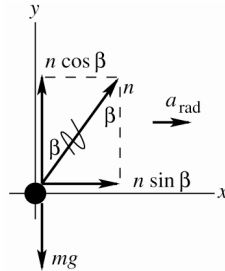


Figure 5.115b

EXECUTE:

$$\begin{aligned} \sum F_y &= ma_y \\ n \cos \beta - mg &= 0 \\ n &= mg / \cos \beta \\ \sum F_x &= ma_x \\ n \sin \beta &= ma_{\text{rad}} \end{aligned}$$

Combine these two equations to eliminate n :

$$\left(\frac{mg}{\cos \beta} \right) \sin \beta = ma_{\text{rad}}$$

$$\frac{\sin \beta}{\cos \beta} = \frac{a_{\text{rad}}}{g}$$

$a_{\text{rad}} = v^2 / R$ and $v = 2\pi R / T$, so $a_{\text{rad}} = 4\pi^2 R / T^2$, where T is the time for one revolution.

$$R = r \sin \beta, \text{ so } a_{\text{rad}} = \frac{4\pi^2 r \sin \beta}{T^2}$$

$$\text{Use this in the above equation: } \frac{\sin \beta}{\cos \beta} = \frac{4\pi^2 r \sin \beta}{T^2 g}$$

This equation is satisfied by $\sin \beta = 0$, so $\beta = 0$, or by

$$\frac{1}{\cos \beta} = \frac{4\pi^2 r}{T^2 g}, \text{ which gives } \cos \beta = \frac{T^2 g}{4\pi^2 r}$$

(a) 4.00 rev/s implies $T = (1/4.00) \text{ s} = 0.250 \text{ s}$

$$\text{Then } \cos \beta = \frac{(0.250 \text{ s})^2 (9.80 \text{ m/s}^2)}{4\pi^2 (0.100 \text{ m})} \text{ and } \beta = 81.1^\circ.$$

(b) This would mean $\beta = 90^\circ$. But $\cos 90^\circ = 0$, so this requires $T \rightarrow 0$. So β approaches 90° as the hoop rotates very fast, but $\beta = 90^\circ$ is not possible.

(c) 1.00 rev/s implies $T = 1.00 \text{ s}$

$$\text{The } \cos \beta = \frac{T^2 g}{4\pi^2 r} \text{ equation then says } \cos \beta = \frac{(1.00 \text{ s})^2 (9.80 \text{ m/s}^2)}{4\pi^2 (0.100 \text{ m})} = 2.48, \text{ which is not possible. The only way to}$$

have the $\sum \vec{F} = m\vec{a}$ equations satisfied is for $\sin \beta = 0$. This means $\beta = 0$; the bead sits at the bottom of the hoop.

EVALUATE: $\beta \rightarrow 90^\circ$ as $T \rightarrow 0$ (hoop moves faster). The largest value T can have is given by $T^2 g / (4\pi^2 r) = 1$ so $T = 2\pi \sqrt{r/g} = 0.635 \text{ s}$. This corresponds to a rotation rate of $(1/0.635) \text{ rev/s} = 1.58 \text{ rev/s}$. For a rotation rate less than 1.58 rev/s, $\beta = 0$ is the only solution and the bead sits at the bottom of the hoop. Part (c) is an example of this.

5.127. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the ball at each position.

SET UP: When the ball is at rest, $a = 0$. When the ball is swinging in an arc it has acceleration component

$$a_{\text{rad}} = \frac{v^2}{R}, \text{ directed inward.}$$

EXECUTE: Before the horizontal string is cut, the ball is in equilibrium, and the vertical component of the tension force must balance the weight, so $T_A \cos \beta = w$ or $T_A = w / \cos \beta$. At point B , the ball is not in equilibrium; its speed is instantaneously 0, so there is no radial acceleration, and the tension force must balance the radial component of the weight, so $T_B = w \cos \beta$ and the ratio $(T_B / T_A) = \cos^2 \beta$.

EVALUATE: At point B the net force on the ball is not zero; the ball has a tangential acceleration.

