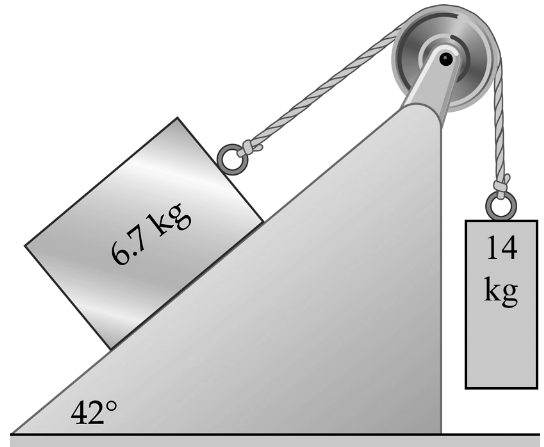
Quiz 3, Phyx 135-1, Fall 2009, Solutions

1) A mass of 14 kg is connected to a mass of 6.7 kg by a frictionless pulley. The 6.7 kg mass is setting on a ramp angled at 42°. There is friction, and $\mu_k = 0.40$ for the ramp. The 14 kg mass is hanging exactly one meter above the floor. If it is allowed to fall, how fast will it be moving when it hits the floor?



Solution: There are two ways to solve this problem. By far the easiest is using conservation of energy. We know the larger mass will *lose* $E = mgh = (14 \text{ kg})(9.8 \text{ m/s}^2)(1 \text{ m}) = 137.2 \text{ J}$ of gravitational energy as it falls. The smaller mass will *gain* $E = mgh = mgd \sin\theta = (6.7 \text{ kg})(9.8 \text{ m/s}^2)(1 \text{ m})(0.66913) = 43.9 \text{ J}$ of gravitational energy as it rises. We also have $E = Fd = \mu Nd = \mu mgd \cos\theta = (0.4)(6.7 \text{ kg})(9.8 \text{ m/s}^2)(1 \text{ m})(0.7431) = 19.5 \text{ J}$ of heat energy generated (from friction) as the 6.7 kg mass moves along the ramp. This leaves us with $137.2 - 43.9 - 19.5 = 73.8 \text{ J}$. The left-over energy can only be kinetic energy, so $73.8 \text{ J} = ½ (6.7 \text{ kg} + 14 \text{ kg}) v^2$, or $v = 2.67 \text{ m/s}$.

The much harder way is to use Newton's Laws and find the acceleration of the system. Since the velocity of the mass as it reaches the ground is what we really want, we must invoke $d = ½ at^2$ and $v = at$, then substitute $t = v/a$ into the first equation to get $d = ½ v^2/a$, or $v = (2da)^{1/2}$. We already know d, so we can readily get v if we can calculate a.

For this particular system, which includes friction, we have no choice but to apply Newton's Laws to each mass separately. For the 6.7 kg mass, the forces are: (1) gravity, pulling down the ramp. (2) friction, and since the block is moving up the ramp, the frictional force points down the ramp. (3) the tension in the cord, which is pulling up the ramp. Writing all this out as an equation: $-mg \sin\theta - \mu mg \cos q + T = ma$, or $-(6.7 \text{ kg})(9.8) \sin(42°) - (0.4)(6.7 \text{ kg})(9.8) \cos(42°) + T = (6.7 \text{ kg})a$, or $T = 6.7\, a + 63.45$.

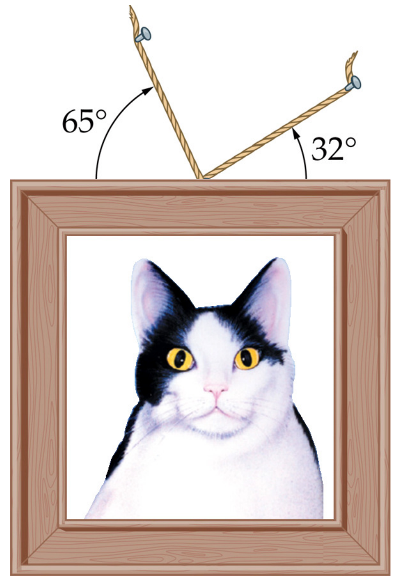
And might I add, if it crossed your mind for a nanosecond that T should equal the weight of the larger mass, just because the larger mass is dangling from the cord, then the ghost of Isaac Newton will return and haunt you this Halloween. The mass is accelerating, so the forces on it *cannot* be equal.

For the 14 kg mass, the forces are just (1) gravity, which is negative, and (2) the tension in the cord, which is positive. *Or are they?* We have already chosen the tension on the 6.7 kg mass to be positive (look at the blue equations), so the positive direction for the cord is *clockwise* as it goes around the pulley. We cannot change this decision now. If clockwise is the "positive" direction for the cord in one place, then clockwise is the "positive" direction everyplace. (Keeping your signs consistent is one of the hardest parts of applying Newton's Laws to systems with several masses in them.) Clockwise motion, for the 14 kg mass, means that "positive" is down! Thus, the gravity on the 14 kg mass is positive, and the tension is negative.

The equation for the 14 kg mass is: $mg - T = ma$, or $(14 \text{ kg})(9.8) - T = (14 \text{ kg})\, a$, or $-T = 14\, a - 137.2$
Adding this equation to the other equation for T yields: $0 = (6.7 + 14)\, a + 63.45 - 137.2$, or $a = 3.56 \text{ m/s}^2$.
Using the equation for the velocity that we previously derived: $v = [(2)(1 \text{ m})(3.56 \text{ m/s}^2)]^{1/2} = 2.67 \text{ m/s}$.

Also, note that $T = 87.3 \text{ N}$ (from substituting $a = 3.56 \text{ m/s}^2$ into either equation for T). This is most certainly not equal to the weight of the 14 kg mass, which is (14)(9.8) = 137.2 N

2) A portrait of Muffy the cat is hanging haphazardly from two strings, as shown. If the portrait has a mass of 4 kg, what is the tension on each of the strings?



Solution: Let T_L and T_R be the tensions for the left and right strings respectively. The y-components of the tensions must equal the portrait's weight, because nothing else is holding the portrait up. We have $T_L \sin(65°) + T_R \sin(32°) = mg$. The x-components of the two tension must equal each other, because the portrait is not moving along the x-axis. We have $T_L \cos(65°) = T_R \cos(32°)$. Inserting numbers gives us $(0.9063)T_L + (0.5299)T_R = (4)(9.8)$, and $(0.4226)T_L = (0.8480)T_R$. The second equation says $T_R = (0.4983)T_L$, and substitution into the first equation gives $(0.9063)T_L + (0.5299)(0.4983)T_L = 39.2$, or $T_L = 33.5$ N. Then $T_R = (33.5)(0.4983) = 16.7$ N.