Here is the problem. A crate with a mass of 300 kg is placed onto a horizontal conveyor belt. The coefficient of friction between the belt and the crate is $\mu = 0.4$, and the belt is moving at a constant speed of 1.2 m/s. The question is, how much energy does the belt (or equivalently, the motor running the belt) need to supply to bring the crate up to 1.2 m/s?

There are two ways to solve this problem: one is very easy algebraically but tricky conceptually, the other somewhat easier conceptually but involved algebraically.

First the easy way, using conservation of energy. We know the crate starts at $v = 0$ and moves to $v = 1.2$ m/s, so we need $E = \frac{1}{2} mv^2 = (0.5)(300 \text{ kg})(1.2 \text{ m/s})^2 = 216$ J of kinetic energy from the belt. But this cannot be the full story, because when the crate hits the belt there has to be some time when the belt is moving faster than the crate. That is, the belt must scrape past the crate and create frictional heat as the crate accelerates. We can deal with this by imagining ourselves to be on the conveyor belt. From that viewpoint, the crate is moving at 1.2 m/s before it reaches the belt, then it slides to zero velocity. The 1.2 m/s of relative velocity translates into 216 J of kinetic energy, all of which must be converted to heat. The belt must supply E = 216 (heat) + 216 (K.E.) = 432 J.

The harder way to solve the problem is with Newton's Laws. When the crate lands on the belt, it will experience a frictional force $F = \mu N = \mu mg = (0.4)(300 \text{ kg})(9.8 \text{ m/s}^2) = 1176$ N. This force will do work on the crate. However, the work done **on** the crate and the work done **by** the belt cannot be the same, because the belt is moving faster than the crate and creating friction. Since we want to know the work done **by** the belt, we have some algebra to do. We will calculate how long it takes the crate to get up to speed and therefore how far it moves, then calculate how far the belt can move in that same time, then calculate the distance that the crate moves *relative* to the belt, then add the frictional heat from this relative motion to the final kinetic energy of the box. (Whew!)

The acceleration on the crate is $a = F/m = (1176 \text{ N})/(300 \text{ kg}) = 3.92 \text{ m/s}^2$.
The time it takes the crate to reach 1.2 m/s is $t = v/a = (1.2 \text{ m/s})/(3.92 \text{ m/s}^2) = 0.3061$ s.
The distance the crate moves while accelerating is $d_C = \frac{1}{2} at^2 = (0.5)(3.92)(0.3061)^2 = 0.1836$ m.
In that same time, the belt will move $d_B = vt = (1.2)(0.3061) = 0.3673$ m.
The distance difference is $0.3673 - 0.1836 = 0.1836$ m. In other words, 0.1836 m of the belt moves past the box while the box is accelerating. This relative motion generates frictional heat = $Fd = (1176 \text{ N})(0.1836 \text{ m}) = 216$ J.
Adding the heat to the box's kinetic energy yields E = 216 J (heat) + 216 J (K.E.) = 432 J, as before.