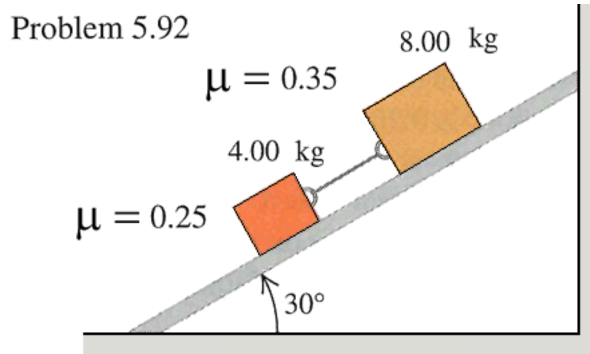
**Problem 5.92** is concerned with the acceleration of two tethered blocks which have different coefficients of friction. Before doing any algebra, you should ask yourself why the cord has any tension on it at all. Aren't both blocks sliding freely under gravity?




Well, yes, but suppose you place a block of sandpaper and a block of ice on a wood ramp. If you release them, the ice will easily out-race the sandpaper to the bottom. Then, if you tie them together, the ice will again race ahead towards the bottom – until it reaches the end of the cord. After making a thump and bouncing around a little, the ice will end up "dangling" from the sandpaper block. The sandpaper acts like an anchor, holding back the ice, whereas the ice becomes an extra force (weight) on the sandpaper, pulling it along a bit faster. This is why the cord has a tension across it. Once the cord is taut, the two blocks must accelerate at the same rate and indeed, it is precisely *because* they are being forced to accelerate together (even though they have different frictional forces) that the cord must exert a tension.

I will solve the problem algebraically. $m_1$ and $\mu_1$ will refer to the lower block, $m_2$ and $\mu_2$ will refer to the upper block, and our positive direction will be down the ramp.

There are three forces acting on block 1: gravity down the ramp, friction opposing the gravity, and the tension in the cord above it. Writing out the acceleration:

$m_1 a = m_1 g \sin\theta - \mu_1 m_1 g \cos\theta - T$ , where we have resolved the gravitational force into the "usual" trigonometric components along the ramp and normal to the ramp. The acceleration equation for block 2 is very similiar: $m_2 a = m_2 g \sin\theta - \mu_2 m_2 g \cos\theta + T$ , the only real difference being the opposite sign on the tension.

To solve for T, we can use a bit of old-school algebra and multiply the first blue equation by $m_2$ and the second one by $m_1$. This yields:

$$m_2 m_1 a = m_2 m_1 g \sin\theta - m_2 \mu_1 m_1 g \cos\theta - m_2 T \quad \text{and}$$

$$m_1 m_2 a = m_1 m_2 g \sin\theta - m_1 \mu_2 m_2 g \cos\theta + m_1 T.$$

Subtracting gives $0 = 0 + (\mu_2 - \mu_1) m_1 m_2 g \cos\theta - (m_1 + m_2)T$, or $T = m_1 m_2 g \cos\theta\, (\mu_2 - \mu_1)/(m_1 + m_2)$

To solve for a, we can just add the original equations to get:
$(m_1 + m_2)a = (m_1 + m_2) g \sin\theta - (\mu_1 m_1 + \mu_2 m_2) g \cos\theta$, or $a = g \sin\theta - g \cos\theta\, (\mu_1 m_1 + \mu_2 m_2)/(m_1 + m_2)$

Inserting the numbers given in the illustration yields:
T = 4 x 8 x 9.8 x 0.866 x (0.35 – 0.25)/(4 + 8) = 2.27 N
a = 9.8 x 0.5 – 9.8 x 0.866 x (0.25 x 4 + 0.35 x 8)/(4 + 8) = 2.21 m/s$^2$